Basic Mathematics

Prelim Question Paper Solution

Time: 3 Hrs.] **[Marks : 100**

Q.1 Attempt any TEN of the following : **[20]**

Q.1(a) Find the value of $\begin{vmatrix} 2 & 3 & 5 \\ 1 & 4 & 2 \\ 3 & 1 & 6 \end{vmatrix}$ **[2]**

(A) $\begin{vmatrix} 2 & 3 & 5 \\ 1 & 4 & 2 \\ 3 & 1 & 6 \end{vmatrix} = 2(24 - 2) - 3(6-6) + 5(1 - 12)$ [1 mark]

$= -11$ [1 mark]

Q.1(b) If $A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$ find 3A – 2B. **[2]**

(A) $3A = 3\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 12 & 21 \end{bmatrix}$ [½ mark]

$2B = 2\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 8 & 12 \end{bmatrix}$ [½ mark]

$\therefore\ 3A - 2B = \begin{bmatrix} 6 & 9 \\ 12 & 21 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ 8 & 12 \end{bmatrix}$ [½ mark]

$= \begin{bmatrix} 4 & 3 \\ 4 & 9 \end{bmatrix}$ [½ mark]

Q.1(c) Find the value of a and b if $\begin{bmatrix} a - 4b & 5 \\ 6 & -a + b \end{bmatrix} = \begin{bmatrix} 11 & 5 \\ 6 & -5 \end{bmatrix}$ **[2]**

(A) $\begin{bmatrix} a - 4b & 5 \\ 6 & -a + b \end{bmatrix} = \begin{bmatrix} 11 & 5 \\ 6 & -5 \end{bmatrix}$

$\therefore\ a - 4b = 11$ [½ mark]

$-a + b = -5$ [½ mark]

$\therefore\ -3b = 6$

$\therefore\ b = -2$ [½ mark]

$\therefore\ a = 3$ [½ mark]

Q.1(d) $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ verify that $(A + B)^T = A^T + B^T$. **[2]**

(A) $A + B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 3 & 2 \\ 5 & 6 \end{bmatrix}$ [½ mark]

$\therefore\ (A + B)^T = \begin{bmatrix} 3 & 5 \\ 2 & 6 \end{bmatrix}$ … (i) [½ mark]

$\therefore\ A^T + B^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 3 & 5 \\ 2 & 6 \end{bmatrix}$ … (ii) [½ mark]

$\therefore$ by (i) and (ii),

$(A + B)^T = A^T + B^T$ [½ mark]

Q.1(e) Resolve into the partial fractions : $\frac{x}{x^2 - x - 2}$ **[2]**

(A) $\frac{x}{x^2 - x - 2} = \frac{x}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$

$\therefore$ x = (x + 1)A + (x – 2)B

Put x – 2 = 0 i.e., x = 2

$\therefore$ 2 = (2 + 1)A + 0

$\therefore$ 2 = 3A

$\therefore$ $\frac{2}{3} = A$ [1 mark]

Put x + 1 = 0 i.e., x = –1

$\therefore$ –1 = 0 + (–1 – 2)B

$\therefore$ –1 = –3B

$\therefore$ $\frac{1}{3} = B$ [½ mark]

$\therefore$ $\frac{x}{x^2 - x - 2} = \frac{\frac{2}{3}}{x-2} + \frac{\frac{1}{3}}{x+1}$ [½ mark]

Q.1(f) Without using calculator find the value of sin (– 330°). **[2]**

(A) sin(–330°) = – sin 330°

= – sin (360°– 30°) [1 mark]

= – sin (– 30°) [1 mark]

= sin (30°) [1 mark]

= $\frac{1}{2}$ or 0.5 [1 mark]

Q.1(g) Prove that : cos 2A = 2 $\cos^2 A$ – 1. **[2]**

(A) cos 2A = cos (A + A)

= cosA cosA – sinA sinA [½ mark]

= $\cos^2 A - \sin^2 A$ [½ mark]

= $\cos^2 A - (1 - \cos^2 A)$ [½ mark]

= $\cos^2 A - 1 + \cos^2 A$ [½ mark]

= $2\cos^2 A - 1$

Q.1(h) If sin A = $\frac{1}{2}$ find sin 3A. **[2]**

(A) sin 3A = 3sinA – $4\sin^3 A$ [1 mark]

= $3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3$ [½ mark]

= 1 [½ mark]

Q.1(i) Prove that : $\frac{\cos 3\theta}{\cos\theta} + \frac{\sin 3\theta}{\sin\theta} = 4 \cos 2\theta$ **[2]**

(A) $\frac{\cos 3\theta}{\cos\theta} + \frac{\sin 3\theta}{\sin\theta} = \frac{\sin\theta\cos 3\theta + \cos\theta\sin 3\theta}{\cos\theta\sin\theta}$ [½ mark]

$= \frac{\sin(\theta + 3\theta)}{\cos\theta\sin\theta}$

$= \frac{\sin 4\theta}{\cos\theta\sin\theta}$ [½ mark]

$= \frac{\sin 2(2\theta)}{\cos\theta\sin\theta}$

$= \dfrac{2\sin 2\theta \cos 2\theta}{\cos\theta \sin\theta}$ [½ mark]

$= \dfrac{2 \cdot 2\sin\theta\cos\theta \cdot \cos 2\theta}{\cos\theta \sin\theta}$

$= 4\cos 2\theta$ [½ mark]

Q.1(j) Prove that $\cos^{-1}(-x) = \pi - \cos^{-1}x$. **[2]**

(A) Let $\cos^{-1}(x) = \theta$

$\therefore \quad x = \cos\theta$ [½ mark]

$\therefore \quad -x = -\cos\theta$

$\therefore \quad -x = \cos(\pi - \theta)$ [½ mark]

$\therefore \quad \cos^{-1}(-x) = \pi - \theta$ [½ mark]

$= \pi - \cos^{-1}x$ [½ mark]

Q.1(k) Find the slope and y-intercept of line $\dfrac{x}{4} - \dfrac{y}{3} = 2$ **[2]**

(A) $\dfrac{x}{4} - \dfrac{y}{3} - 2 = 0$

$\therefore \quad a = \dfrac{1}{4} \qquad b = -\dfrac{1}{3} \qquad c = -2$

$\therefore$ slope $m = -\dfrac{a}{b} = -\dfrac{\frac{1}{4}}{\frac{-1}{3}} = \dfrac{3}{4}$ or 0.75 [1 mark]

y-int $= -\dfrac{c}{b} = \dfrac{-2}{\frac{-1}{3}} = -6$ [1 mark]

Q.1(l) Find the range and the coefficient of range for the following data :
120, 100, 130, 50, 150 **[2]**

(A) Smallest Value S = 50, Largest Value L = 150

$\therefore$ Range $= L - S = 150 - 50$

$= 100$ [1 mark]

Coeff. of Range $= \dfrac{L - S}{L + S} = \dfrac{150 - 50}{150 + 50}$ [½ mark]

$= \dfrac{1}{2}$ or 0.5 [½ mark]

Q.2 Attempt any FOUR of the following. **[16]**

Q.2(a) Solve the equations, for y and z **[4]**

$\dfrac{x}{4} - \dfrac{y}{3} + \dfrac{z}{2} = 5,\ \dfrac{x}{3} + \dfrac{y}{2} - \dfrac{z}{5} = 11,\ \dfrac{x}{7} - \dfrac{y}{9} + \dfrac{z}{6} = -2$, by using Cramer's rule.

(A) $\dfrac{x}{4} - \dfrac{y}{3} + \dfrac{z}{2} = 5 \qquad \dfrac{x}{3} + \dfrac{y}{2} - \dfrac{z}{5} = 11 \qquad \dfrac{x}{7} - \dfrac{y}{9} + \dfrac{z}{6} = -2$

$$\therefore \quad D = \begin{vmatrix} \frac{1}{4} & -\frac{1}{3} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & -\frac{1}{5} \\ \frac{1}{7} & -\frac{1}{9} & \frac{1}{6} \end{vmatrix} = \frac{1}{4}\left(\frac{1}{12} - \frac{1}{45}\right) + \frac{1}{3}\left(\frac{1}{18} + \frac{1}{35}\right) + \frac{1}{2}\left(-\frac{1}{27} - \frac{1}{14}\right)$$

$= -\dfrac{11}{1008}$ or -0.0109 [1 mark]

$$D_y = \begin{vmatrix} \frac{1}{4} & 5 & \frac{1}{2} \\ \frac{1}{3} & 11 & -\frac{1}{5} \\ \frac{1}{7} & -2 & \frac{1}{6} \end{vmatrix} = \frac{1}{4}\left(\frac{11}{6}-\frac{2}{5}\right)-5\left(\frac{1}{18}+\frac{1}{35}\right)+\frac{1}{2}\left(-\frac{2}{3}-\frac{11}{7}\right)$$

$$= -\frac{2977}{2520} \quad \text{or} \quad -1.181$$ [1 mark]

$$D_z = \begin{vmatrix} \frac{1}{4} & -\frac{1}{3} & 5 \\ \frac{1}{3} & \frac{1}{2} & 11 \\ \frac{1}{7} & -\frac{1}{9} & -2 \end{vmatrix} = \frac{1}{4}\left(-1+\frac{11}{9}\right)+\frac{1}{3}\left(-\frac{2}{3}-\frac{11}{7}\right)+5\left(-\frac{1}{27}-\frac{1}{14}\right)$$

$$= -\frac{233}{189} \quad \text{or} \quad -1.233$$ [1 mark]

$$\therefore \quad y = \frac{D_y}{D} = \frac{-1.181}{-0.0109} = 108.254$$ [½ mark]

$$z = \frac{D_z}{D} = \frac{-1.233}{-0.0109} = 112.970$$ [½ mark]

Q.2(b) If $A = \begin{bmatrix} x & 2 & -5 \\ 3 & 1 & 2y \end{bmatrix}$ and $B = \begin{bmatrix} 2y+5 & 6 & -15 \\ 9 & 3 & -6 \end{bmatrix}$ and if 3A = B, find x, y. **[4]**

(A) Given 3A = B

$$\therefore \quad 3\begin{bmatrix} x & 2 & -5 \\ 3 & 1 & 2y \end{bmatrix} = \begin{bmatrix} 2y+5 & 6 & -15 \\ 9 & 3 & -6 \end{bmatrix}$$ [1 mark]

$$\therefore \quad 3\begin{bmatrix} 3x & 6 & -15 \\ 9 & 3 & 6y \end{bmatrix} = \begin{bmatrix} 2y+5 & 6 & -15 \\ 9 & 3 & -6 \end{bmatrix}$$ [1 mark]

$\therefore \quad 3x = 2y + 5 \quad$ and $\quad 6y = -6$ [1 mark]

$\therefore \quad x = 1 \quad$ and $\quad y = -1$ [1 mark]

Q.2(c) If $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$, $C = \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$ **[4]**

verify that A (B + C) = AB + BC.

(A)

$$B + C = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 \\ 4 & 3 \end{bmatrix}$$ [1 mark]

$$\therefore \quad A(B+C) = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} -1+8 & 2+6 \\ 2+12 & -4+9 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 8 \\ 14 & 5 \end{bmatrix}$$ [1 mark]

$$AB = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2+4 & 1+6 \\ -4+6 & -2+9 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 7 \\ 2 & 7 \end{bmatrix}$$ [½ mark]

$$AC = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -3+4 & 1+0 \\ 6+6 & -2+0 \end{bmatrix}$$

$= \begin{bmatrix} 1 & 1 \\ 12 & -2 \end{bmatrix}$ [½ mark]

$\therefore \quad AB + AC = \begin{bmatrix} 6 & 7 \\ 2 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 12 & -2 \end{bmatrix}$

$= \begin{bmatrix} 7 & 8 \\ 14 & 5 \end{bmatrix}$ [½ mark]

$\therefore \quad A(B + C) = AB + AC$ [½ mark]

Q.2(d) Using matrix inversion method, solve the equations. **[4]**

$5x + y = 13,\ 3x + 2y = 5$

(A) $5x + y = 13$

$3x + 2y = 5$

$\therefore \quad A = \begin{bmatrix} 5 & 1 \\ 3 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$

$\therefore \quad |A| = \begin{vmatrix} 5 & 1 \\ 3 & 2 \end{vmatrix} = 10 - 3 = 7$ [1 mark]

$C(A) = \begin{bmatrix} 2 & -3 \\ -1 & 5 \end{bmatrix}$ [1 mark]

$\therefore \quad adj(A) = \begin{bmatrix} 2 & -1 \\ -3 & 5 \end{bmatrix}$ [½ mark]

$\therefore \quad A^{-1} = \frac{1}{|A|} adj(A)$

$= \frac{1}{7}\begin{bmatrix} 2 & -1 \\ -3 & 5 \end{bmatrix}$ [½ mark]

$\therefore$ the solution is,

$X = A^{-1}B$

$= \frac{1}{7}\begin{bmatrix} 2 & -1 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 13 \\ 5 \end{bmatrix}$

$= \frac{1}{7}\begin{bmatrix} 21 \\ -14 \end{bmatrix}$

$= \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

$\therefore \quad x = 3, y = -2$ [1 mark]

Q.2(e) Resolve into the partial fractions : $\frac{x^2 + 1}{2x^4 + 5x^2 + 2}$ **[4]**

(A) $\frac{x^2 + 1}{2x^4 + 5x^2 + 2}$ (Put $x^2 = y$)

$= \frac{y + 1}{2y^2 + 5y + 2}$ [1 mark]

$= \frac{y + 1}{(2y + 1)(y + 2)} = \frac{A}{2y + 1} + \frac{B}{y + 2}$

$\therefore \quad y + 1 = (y + 2)A + (2y + 1)B$

Put $2y + 1 = 0$ or $y = -\frac{1}{2}$

$\therefore \quad -\frac{1}{2} + 1 = \left(-\frac{1}{2} + 2\right)A + 0$

$\therefore \quad \frac{1}{2} = \frac{3}{2}A$

$\therefore \quad \frac{1}{3} = A$ [1 mark]

Put y + 2 = 0 or y = –2

$\therefore \quad -2 + 1 = 0 + (-4 + 1)B$

$\therefore \quad -1 = -3B$

$\therefore \quad \frac{1}{3} = B$ [1 mark]

$\therefore \quad \frac{y+1}{2y^2+5y+2} = \frac{\frac{1}{3}}{2y+1} + \frac{\frac{1}{3}}{y+2}$ [½ mark]

$\therefore \quad \frac{x^2+1}{2x^4+5x^2+2} = \frac{\frac{1}{3}}{2x^2+1} + \frac{\frac{1}{3}}{x^2+2}$ [½ mark]

Q.2(f) Resolve into partial fractions $\frac{x^2+23x}{(x+3)(x^2+1)}$ **[4]**

(A) $\frac{x^2+23x}{(x+3)(x^2+1)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+1}$

$\therefore \quad x^2 + 23x = (x + 3)(x^2 + 1)\left[\frac{A}{x+3} + \frac{Bx+C}{x^2+1}\right]$

$\therefore \quad x^2 + 23x = (x^2 + 1)A + (x + 3)(Bx + C)$

Put x = – 3

$\therefore \quad (-3)^2 + 23(-3) = ((-3)^2 + 1)A + 0$

$\therefore \quad -60 = 10A$

$\therefore \quad -6 = A$ [1 mark]

Put x = 0

$\therefore \quad 0^2 + 23(0) = (0^2 + 1)A + (0 + 3)(0 + C)$

$\therefore \quad 0 = A + 3C$

$\therefore \quad 0 = -6 + 3C$

$\therefore \quad 6 = 3C$

$\therefore \quad 2 = C$

Put x = 1

$\therefore \quad 1^2 + 23(1) = (1^2 + 1)A + (1 + 3)(B + C)$

$\therefore \quad 24 = 2A + 4B + 4C$

$\therefore \quad 24 = 2(-6) + 4B + 4(2)$

$\therefore \quad 28 = 4B$

$\therefore \quad 7 = B$

$\therefore \quad \frac{x^2+23x}{(x+3)(x^2+1)} = \frac{-6}{x+3} + \frac{7x+2}{x^2+1}$ [1 mark]

Q.3 Attempt any FOUR of the following. **[16]**

Q.3(a) Solve the equations x + 2y + 3z = 1, 2x + 3y + 2z = 2 and 3x + 2y + 4z = 1 by using matrix inversion method. **[4]**

(A) x + 2y + 3z = 1,

2x + 3y + 2z = 2,

3x + 2y + 4z = 1

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 2 & 4 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad K = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$\therefore \quad |A| = 1(12 - 4) - 2(8 - 6) + 3(2 - 9) = -11$ [1 mark]

The minors of matrix A are

$A_{11} = \begin{vmatrix} 3 & 2 \\ 2 & 4 \end{vmatrix} = 8$ $\quad A_{12} = -\begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} = -2$ $\quad A_{13} = \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = -5$

$A_{21} = -\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = -2$ $\quad A_{22} = \begin{vmatrix} 1 & 3 \\ 3 & 4 \end{vmatrix} = -5$ $\quad A_{23} = -\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = 4$ [1 mark]

$A_{31} = \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = -5$ $\quad A_{32} = -\begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} = 4$ $\quad A_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1$

$\therefore$ the matrix of cofactors is,

$$\therefore \quad C(A) = \begin{bmatrix} 8 & -2 & -5 \\ -2 & -5 & 4 \\ -5 & 4 & -1 \end{bmatrix}$$ [½ mark]

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} 8 & -2 & -5 \\ -2 & -5 & 4 \\ -5 & 4 & -1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{-11}\begin{bmatrix} 8 & -2 & -5 \\ -2 & -5 & 4 \\ -5 & 4 & -1 \end{bmatrix}$$ [½ mark]

$$\therefore \quad X = A^{-1}K = \frac{1}{-11}\begin{bmatrix} 8 & -2 & -5 \\ -2 & -5 & 4 \\ -5 & 4 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} -1 \\ -8 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{11} \\ \frac{8}{11} \\ -\frac{2}{11} \end{bmatrix}$$ [½ mark]

$\therefore \quad x = \frac{1}{11} \quad y = \frac{8}{11} \quad z = -\frac{2}{11}$

Q.3(b) Resolve into partial fractions $\frac{x^2}{(x^2+1)(x^2+2)}$. **[4]**

(A) Put $x^2 = y$

$$\frac{x^2}{(x^2+1)(x^2+2)} = \frac{y}{(y+1)(y+2)} = \frac{A}{y+1} + \frac{B}{y+2}$$

$\therefore \quad y = (y+1)(y+2)\left[\frac{A}{y+1} + \frac{B}{y+2}\right]$

$\therefore \quad y = (y+2)A + (y+1)B$

Put $y = -1$

$\therefore \quad -1 = (-1+2)A + 0$

$\therefore \quad -1 = A$ [1 mark]

Put $y = -2$

$\therefore \quad -2 = 0 + (-2+1)B$

$\therefore \quad -2 = -B$

$\therefore \quad 2 = B$ [1 mark]

$$\therefore \quad \frac{y}{(y+1)(y+2)} = \frac{-1}{y+1} + \frac{2}{y+2}$$ [1 mark]

$$\therefore \quad \frac{x^2}{(x^2+1)(x^2+2)} = \frac{-1}{x^2+1} + \frac{2}{x^2+2}$$ [1 mark]

Q.3(c) Resolve into the partial fractions : $\dfrac{e^x+1}{2e^{2x}+7e^x+5}$ **[4]**

(A) $\dfrac{e^x+1}{2e^{2x}+7e^x+5}$ (Put $e^x = y$)

$= \dfrac{y+1}{2y^2+7y+5}$ [1 mark]

$= \dfrac{y+1}{(2y+5)(y+1)}$ [1 mark]

$= \dfrac{1}{2y+5}$ [1 mark]

$= \dfrac{1}{2e^x+5}$ [1 mark]

Q.3(d) Prove that $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A\cdot\tan B}$ **[4]**

(A) $\tan(A-B) = \dfrac{\sin(A-B)}{\cos(A-B)}$

$= \dfrac{\sin A\cos B-\cos A\sin B}{\cos A\cos B+\sin A\sin B}$ [1 mark]

$= \dfrac{\dfrac{\sin A\cos B-\cos A\sin B}{\cos A\cos B}}{\dfrac{\cos A\cos B+\sin A\sin B}{\cos A\cos B}}$ [1 mark]

$= \dfrac{\dfrac{\sin A\cos B}{\cos A\cos B}-\dfrac{\cos A\sin B}{\cos A\cos B}}{\dfrac{\cos A\cos B}{\cos A\cos B}+\dfrac{\sin A\cos B}{\cos A\cos B}}$ [1 mark]

$= \dfrac{\tan A-\tan B}{1+\tan A\tan B}$ [1 mark]

Q.3(e) Prove that : $2\cot^{-1}(3) + \operatorname{cosec}^{-1}\left(\dfrac{5}{4}\right) = \dfrac{\pi}{2}$. **[4]**

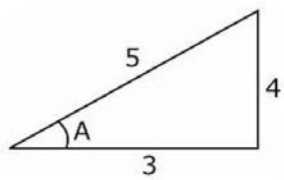
(A) $2\cot^{-1}(3) = \tan^{-1}\left(\dfrac{1}{3}\right)+\tan^{-1}\left(\dfrac{1}{3}\right)$

$= \tan^{-1}\left(\dfrac{\dfrac{1}{3}+\dfrac{1}{3}}{1-\left(\dfrac{1}{3}\right)\left(\dfrac{1}{3}\right)}\right)$ [1 mark]

$= \tan^{-1}\left(\dfrac{3}{4}\right)$ [1 mark]

Let $A = \operatorname{cosec}^{-1}\left(\dfrac{5}{4}\right)$

$\therefore\ \operatorname{cosec}A = \dfrac{5}{4}$



$\therefore\ 2\cot^{-1}(3) + \operatorname{cosec}^{-1}\left(\dfrac{5}{4}\right) = \tan^{-1}\left(\dfrac{3}{4}\right)+\tan^{-1}\left(\dfrac{4}{3}\right)$

$= \tan^{-1}\left(\dfrac{\dfrac{3}{4}+\dfrac{4}{3}}{1-\left(\dfrac{3}{4}\right)\left(\dfrac{4}{3}\right)}\right)$ [1 mark]

$= \tan^{-1}(\infty)$

$= \dfrac{\pi}{2}$ [1 mark]

Q.3(f) Prove that $\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right) = \cot^{-1}\left(\frac{9}{2}\right)$. **[4]**

(A) $\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right) = \tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{13}}{1-\frac{1}{7}\cdot\frac{1}{13}}\right)$ [2 marks]

$= \tan^{-1}\left(\frac{20}{90}\right)$

$= \tan^{-1}\left(\frac{2}{9}\right)$ [1 mark]

$= \cot^{-1}\left(\frac{9}{2}\right)$ [1 mark]

Q.4 Attempt any FOUR of the following. **[16]**

Q.4(a) Without using the calculator find the value of : **[4]**

$$\frac{4}{3\tan 30^\circ}+3\sin^2 120^\circ = \text{cosec}^2 3\theta - \frac{3}{4\cot^2 120^\circ}+\cos^2 270^\circ$$

(A) $\tan^2 30^\circ = \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$ [½ mark]

$\sin 120^\circ = \sin(90^\circ + 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$ [½ mark]

$\therefore \sin^2 120^\circ = \frac{3}{4}$ [½ mark]

$\text{cosec} 30^\circ = 2$

$\therefore \text{cosec}^2 30^\circ = 4$ [½ mark]

$\cot 120^\circ = \cot(90^\circ + 30^\circ)$

$= -\tan 30^\circ$

$= -\frac{1}{\sqrt{3}}$

$\therefore \cot^2 120^\circ = \frac{1}{3}$ [½ mark]

$\therefore \cos 270^\circ = \cos(3 \times 90^\circ + 0)$

$= \sin 0$

$= 0$

$\therefore \cos^2 270^\circ = 0$

But given that

$$\frac{4}{3\tan^2 30^\circ}+3\sin^2 120^\circ - \text{cosec}^2 30^\circ - \frac{3}{4\cot^2 120^\circ}+\cos^2 270^\circ$$

$= \frac{4}{3\left(\frac{1}{3}\right)}+3\left(\frac{3}{4}\right)-4\frac{3}{4\left(\frac{1}{3}\right)}+0$ [½ mark]

$= \frac{9}{2}$ or 4.5 [½ mark]

Q.4(b) Prove that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **[4]**

(A) $\cos 3\theta = \cos(\theta + 2\theta)$

$= \cos\theta \cos 2\theta - \sin\theta \sin 2\theta$ [1 mark]

$= \cos\theta (2\cos^2\theta - 1) - \sin\theta \cdot 2\sin\theta\cos\theta$ [1 mark]

$= \cos\theta \cdot (2\cos^2\theta - 1) - 2\sin^2\theta \cdot \cos\theta$

$= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta)\cos\theta$ [1 mark]

$= 4\cos^3\theta - 3\cos\theta$ [1 mark]

Q.4(c) Prove that : tan A + tan B + tan C = tan A ·tan B·tan C **[4]**

(A) We have, $A + B + C = 180°$ or π

$\therefore \quad A + B = 180° - C$

$\therefore \quad \tan(A + B) = \tan(180° - C)$ [1 mark]

$\therefore \quad \dfrac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$ [1 mark]

$\therefore \quad \tan A + \tan B = -\tan C\,[1 - \tan A \tan B]$

$\therefore \quad \tan A + \tan B = -\tan C + \tan A \tan B \tan C$ [1 mark]

$\therefore \quad \tan A + \tan B + \tan C = \tan A \tan B \tan C$ [1 mark]

Q.4(d) Prove that $\sin20°\sin40°\sin60°\sin80° = \dfrac{3}{16}$. **[4]**

(A) $\sin20°\sin40°\sin60°\sin80° = \sin 20°\sin40° \dfrac{\sqrt{3}}{2}\sin80°$ [½ mark]

$= \dfrac{\sqrt{3}}{2}\cdot\dfrac{-1}{2}(-2\sin 20°\sin40°)\sin80°$

$= \dfrac{\sqrt{3}}{4}(\cos 60° - \cos 20°)\sin80°$ [½ mark]

$= -\dfrac{\sqrt{3}}{4}\left(\dfrac{1}{2} - \cos 20°\right)\sin 80°$ [½ mark]

$= -\dfrac{\sqrt{3}}{4}\left(\dfrac{1}{2}\sin 80° - \sin 80°\cos 20°\right)$

$= -\dfrac{\sqrt{3}}{4}\left(\dfrac{1}{2}\sin 80° - \dfrac{1}{2}\cdot 2\sin 80°\cos 20°\right)$

$= -\dfrac{\sqrt{3}}{4}\cdot\dfrac{1}{2}[\sin80° - (\sin100° + \sin60°)]$ [½ mark]

$= -\dfrac{\sqrt{3}}{8}\left[\sin 80° - \sin 100° - \dfrac{\sqrt{3}}{2}\right]$ [½ mark]

$= -\dfrac{\sqrt{3}}{8}\left[2\cos 90°\sin 20° - \dfrac{\sqrt{3}}{2}\right]$ [½ mark]

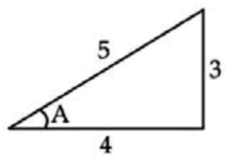
$= -\dfrac{\sqrt{3}}{8}\left[0 - \dfrac{\sqrt{3}}{2}\right]$ [½ mark]

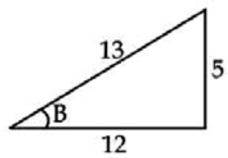
$= \dfrac{3}{16}$ [½ mark]

Q.4(e) Prove that : $\cos^{-1}\left(\dfrac{4}{5}\right) + \cos^{-1}\left(\dfrac{12}{13}\right) = \cos^{-1}\left(\dfrac{33}{65}\right)$. **[4]**

(A) $A = \cos^{-1}\left(\dfrac{4}{5}\right)$ $\qquad B = \cos^{-1}\left(\dfrac{12}{13}\right)$

$\therefore \quad \cos A = \dfrac{4}{5}$ $\qquad \cos B = \dfrac{12}{13}$





$\cos(A + B) = \cos A \cos B - \sin A \sin B$ [1 mark]

$= \dfrac{4}{5}\times\dfrac{12}{13} - \dfrac{3}{5}\times\dfrac{5}{13}$ [1 mark]

$= \dfrac{33}{65}$ [1 mark]

$\therefore \quad A + B = \cos^{-1}\left(\dfrac{33}{65}\right)$ [½ mark]

$\therefore \quad \cos^{-1}\left(\dfrac{4}{5}\right) + \cos^{-1}\left(\dfrac{12}{13}\right) = \cos^{-1}\left(\dfrac{33}{65}\right)$ [½ mark]

Q.4(f) Prove that $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$. **[4]**

(A) $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi + \tan^{-1}\left(\frac{1+2}{1-1\cdot 2}\right) + \tan^{-1}(3)$ [1 mark]

$= \pi + \tan^{-1}(-3) + \tan^{-1}(3)$ [1 mark]

$= \pi - \tan^{-1}(3) + \tan^{-1}(3)$ [1 mark]

$= \pi$ [1 mark]

Q.5 Attempt any FOUR of the following. **[16]**

Q.5(a) Without using calculator prove that : **[4]**

$\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$

(A) $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \cos 20^\circ \cos 40^\circ \left(\frac{1}{2}\right)\cos 80^\circ$ [½ mark]

$= \frac{1}{2}\cdot\frac{1}{2}(2\cos 20^\circ \cos 40^\circ)\cos 80^\circ$

$= \frac{1}{4}(\cos 60^\circ + \cos 20^\circ)\cos 80^\circ$ [½ mark]

$= \frac{1}{4}\left(\frac{1}{2} + \cos 20^\circ\right)\cos 80^\circ$ [½ mark]

$= \frac{1}{4}\left(\frac{1}{2}\cos 80^\circ + \cos 80^\circ \cos 20^\circ\right)$

$= \frac{1}{4}\left(\frac{1}{2}\cos 80^\circ + \frac{1}{2}\cdot 2\cos 80^\circ \cos 20^\circ\right)$

$= \frac{1}{4}\cdot\frac{1}{2}[\cos 80^\circ + (\cos 100^\circ + \cos 60^\circ)]$ [½ mark]

$= \frac{1}{8}\left[\cos 80^\circ + \cos 100^\circ + \frac{1}{2}\right]$ [½ mark]

$= \frac{1}{8}\left[2\cos 90^\circ \cos(-10^\circ) + \frac{1}{2}\right]$ [½ mark]

$= \frac{1}{8}\left[0 + \frac{1}{2}\right]$ [½ mark]

$= \frac{1}{16}$ [½ mark]

Q.5(b) Prove that $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$ **[4]**

(A) We know that,

$2\sin A\cos B = \sin(A + B) + \sin(A - B)$ [1 mark]

Put $A + B = C$

$A - B = D$ [1 mark]

$\therefore \quad A = \frac{C+D}{2}$ and $B = \frac{C-D}{2}$ [1 mark]

$\therefore \quad \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$ [1 mark]

Q.5(c) Prove that : $\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left[\frac{x+y}{1-xy}\right]$ $x > 0, y > 0, xy < 1$. **[4]**

(A) Put $\tan^{-1} x = A$ and $\tan^{-1} y = B$

$\therefore \quad x = \tan A$ and $y = \tan B$

$\therefore \quad \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ [1 mark]

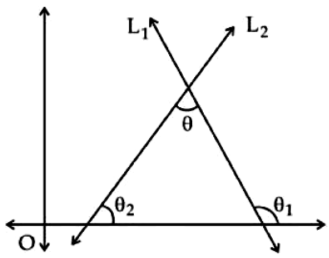
$= \dfrac{x+y}{1-xy}$ [1 mark]

$\therefore \quad A + B = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)$ [1 mark]

$\therefore \quad \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)$ [1 mark]

Q.5(d) Prove that if θ is the acute angle between the lines with slopes m_1 and m_2 then $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$. **[4]**

(A)



[½ mark]

Let θ_1 = Angle of inclination of L_1

θ_2 = Angle of inclination of L_2

$\therefore$ Slope of L_1 is $m_1 = \tan\theta_1$

Slope of L_2 is $m_2 = \tan\theta_2$ [½ mark]

$\therefore$ from figure,

$\theta = \theta_1 - \theta_2$

$\therefore \quad \tan\theta = \tan(\theta_1 - \theta_2)$

$= \dfrac{\tan(\theta_1) - \tan(\theta_2)}{1 + \tan(\theta_1)\tan(\theta_2)}$ [1 mark]

$= \dfrac{m_1 - m_2}{1 + m_1 \cdot m_2}$ [1 mark]

For angle to be acute,

$\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 \cdot m_2}\right|$ [1 mark]

Q.5(e) Find the equation of the straight line passing through (–3, 10) and sum of their intercept is 8. **[4]**

(A) Let x – int = a   y – int = b

$\therefore \quad a + b = 8$

$\therefore$ equation is

$\dfrac{x}{a} + \dfrac{y}{b} = 1$ or $\dfrac{x}{a} + \dfrac{y}{8-a} = 1$

$\therefore \quad bx + ay = ab$

$\therefore \quad (8 - a)x + ay = a(8 - a)$ [1 mark]

But passing through (–3, 10)

$\therefore \quad -3(8 - a) + 10a = a(8 - a)$ [1 mark]

$\therefore \quad -24 + 3a + 10a = 8a - a^2$

$\therefore \quad a^2 + 5a - 24 = 0$

$\therefore \quad a = 3, -8$ [½ + ½ mark]

$\therefore \quad \dfrac{x}{3} + \dfrac{y}{5} = 1$ or $\dfrac{x}{-8} + \dfrac{y}{16} = 1$ [½ + ½ mark]

Q.5(f) Find the length of the perpendicular from (3, 2) on the line $4x - 6y - 5 = 0$. **[4]**

(A) Given $4x - 6y - 5 = 0$

$\therefore$ $A = 4$, $B = -6$, $C = -5$

$\therefore$ the length of the perpendicular is,

$$p = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$$

$$= \left|\frac{4(3) - 6(2) - 5}{\sqrt{4^2 + (-6)^2}}\right|$$ [2 marks]

$$= \frac{5}{\sqrt{52}} \text{ or } 0.693$$ [1 + 1 mark]

Q.6 Attempt any FOUR of the following. [16]

Q.6(a) Find the equation of straight line passing through (5, 6) and making angle 150° with x-axis. **[4]**

(A) Given $\theta = 150°$

$\therefore$ slope $m = \tan\theta = \tan 150°$ [1 mark]

$= -\frac{1}{\sqrt{3}}$ [1 mark]

$\therefore$ equation is

$y - y_1 = m(x - x_1)$

$\therefore$ $y - 6 = -\frac{1}{\sqrt{3}}(x - 5)$ [1 mark]

$\therefore$ $\sqrt{3}y - 6\sqrt{3} = -x + 5$

$\therefore$ $x + \sqrt{3}y - 6\sqrt{3} - 5 = 0$ [1 mark]

Q.6(b) Find the equation of straight line passing through the points (–4, 6) and (8, –3). **[4]**

(A) $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

$\therefore$ $\frac{y - 6}{-3 - 6} = \frac{x + 4}{8 + 4}$ [1 mark]

$\therefore$ $\frac{y - 6}{-9} = \frac{x + 4}{12}$ [1 mark]

$\therefore$ $12(y - 6) = -9(x + 4)$

$\therefore$ $12y - 72 = -9x - 36$ [1 mark]

$\therefore$ $9x + 12y - 36 = 0$ or $-9x - 12y + 36 = 0$

or $3x + 4y - 12 = 0$ or $-3x - 4y + 12 = 0$ [1 mark]

Q.6(c) The scores of two batsmen A and B in ten innings during a certain season as under : **[4]**

A	32	28	47	63	71	39	10	60	96	14
B	19	31	48	53	67	90	10	62	40	80

Find which of two batsmen is more consisting in scoring (use coefficient of variance).

(A) **For Batsman A :**

x_i	x_i^2
32	1024
28	784
47	2209
63	3969
71	5041
39	1521
10	100
60	3600

96	9216
14	196
460	**27660**

$\bar{x} = \dfrac{460}{10} = 46$ [½ mark]

$\sigma = \sqrt{\dfrac{27660}{10} - \left(\dfrac{460}{10}\right)^2} = 25.495$ [½ mark]

$CV(A) = \dfrac{25.495}{46} \times 100 = 55.424$ [½ mark]

For Batsman B :

x_i	x_i^2
19	361
31	961
48	2304
53	2809
67	4489
90	8100
10	100
62	3844
40	1600
80	6400
500	**30968**

$\bar{x} = \dfrac{500}{10} = 50$ [½ mark]

$\sigma = \sqrt{\dfrac{30968}{10} - \left(\dfrac{500}{10}\right)^2} = 24.429$ [½ mark]

$CV(B) = \dfrac{24.429}{50} \times 100 = 48.858$ [½ mark]

$\therefore$ CV(B) < CV(A) [½ mark]

$\therefore$ B is more consistent. [½ mark]

Q.6(d) Find the S.D. of following data : **[4]**

Class-interval	0-10	10-20	20-30	30-40	40-50
Frequency	3	5	8	3	1

(A)

Class	x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
0-10	5	3	15	25	75
10-20	15	5	75	225	1125
20-30	25	8	200	625	5000
30-40	35	3	105	1225	3675
40-50	45	1	45	2025	2025
		20	**440**		**11900**

[1+1 marks]

$$S.D. = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2}$$

$= \sqrt{\dfrac{11900}{20} - \left(\dfrac{440}{20}\right)^2}$ [1 mark]

$= 10.536$ [1 mark]

Q.6(e) Calculate the mean deviation for the following data : **[4]**

Expenditure (Rs.)	40-59	60-79	80-99	100-119	120-139
No. of families	50	300	500	200	60

(A)

Class	x_i	f_i	f_ix_i	$D_i = \lvert x_i - \bar{x} \rvert$	f_iD_i
40-59	49.5	50	2475	38.3559	1927.95
60-79	69.5	300	20850	18.559	5567.7
80-99	89.5	500	44750	1.441	720.5
100-119	109.5	200	21900	21.441	4288.2
120-139	129.5	60	7770	41.441	2486.46
		1110	**97745**		**14990.81**

[1 + 1 mark]

$$\bar{x} = \frac{\sum f_ix_i}{N} = \frac{97745}{1110} = 88.059$$ [1 mark]

$$\text{M.D.} = \frac{\sum f_iD_i}{N}$$

$$= \frac{14990.81}{1110}$$ [½ mark]

$$= 13.505$$ [½ mark]

Q.6(f) Find variance and coefficient of variance of the following data : **[4]**

Class-interval	0-10	10-20	20-30	30-40	40-50
Frequencies	14	23	27	21	15

(A)

Class	x_i	f_i	f_ix_i	x_i^2	$f_ix_i^2$
0-10	5	14	70	25	350
10-20	15	23	345	225	5175
20-30	25	27	675	625	16875
30-40	35	21	735	1225	25725
40-50	45	15	675	2025	30375
			2500		**78500**

[1 marks]

$$\bar{x} = \frac{\sum f_ix_i}{N} = \frac{2500}{100} = 2.5$$ [1 mark]

$$\text{S.D.} = \sqrt{\frac{\sum f_ix_i^2}{N} - \left(\frac{\sum f_ix_i}{N}\right)^2}$$

$$= \sqrt{\frac{78500}{100} - \left(\frac{2500}{100}\right)^2}$$

$$= 12.649$$ [½ mark]

$\therefore$ Variance $= (\text{S.D.})^2$

$= 12.649^2$

$= 159.997$ [½ mark]

$$\text{Coeff. of Variance} = \frac{\text{S.D.}}{\bar{x}} \times 100$$

$$= \frac{12.649}{25} \times 100$$

$$= 50.596$$ [½ mark]

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