S.Y. Diploma : Sem. IV
[CE/CS/CR/CV]

# Theory of Structures

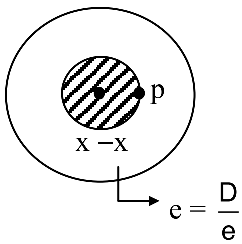
Solution of Prelim Paper

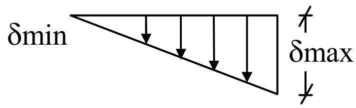
**Q.1(a) Attempt any SIX of the following : [12]**

**Q.1(a) (i) Define "Core of a Section." Sketch resultant stress diagram if load acts on the boundary of core of section. [2]**

**Ans.:** [Definition - 1 mark, Diagram - 1 mark]

**Definition:** The centrally located portion of a section within the load line falls so as to produce only compressive stress is called as core of section.




Resultant stress diagram if load acts on the boundary of core of section.

**Q.1(a) (ii) Write the differential equation for slope and deflection and state terms used in equation. [2]**

**Ans.:** The differential equation for slope

$$\frac{\delta y}{\delta x} = \int \frac{Mx}{Ex}$$

$$Y = \int \frac{\delta y}{\delta x} \cdot \frac{1}{EI}$$

$$\frac{\delta y}{\delta x} = \text{Slope}$$

Mx = loading

Y = deflection [1 mark]

The differential equation for deflection

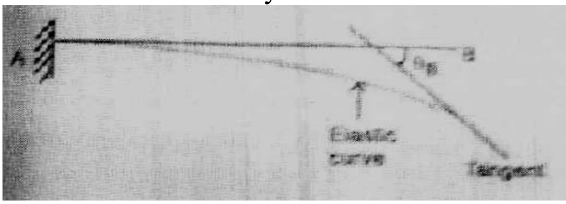
$$y = \iint \frac{Mx}{Ex}$$ [1 mark]

**Q.1(a) (iii) What do you understand by boundary conditions of a beam? State the boundary condition for two different nature of beam support. [2]**

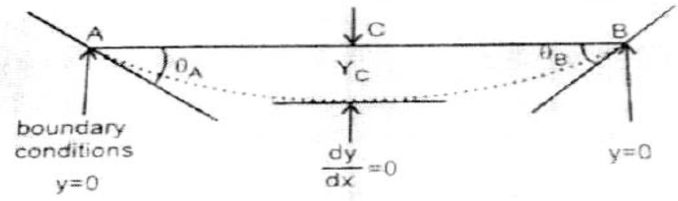
**Ans.:** It is a general mathematical principle that the number of boundary condition necessary to determine a solution to a differential equation matches the order of the differential equation.

- Cantilever beam X is consider from free end
  Boundary condition At $x = 0$ . $y = y\ max$ .$\theta = \theta max$

At $x = L$ . $y = 0$ .$\theta = 0$



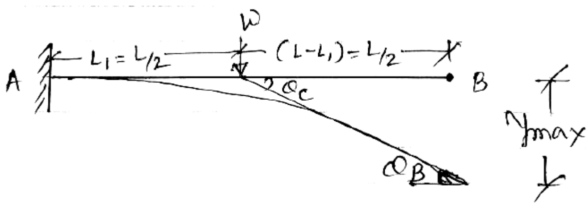

- Simply supported beam The condition to locate point of maximum deflection is slope of tangent at that point is zero
  Boundary condition At $X = 0$ $y = 0$ .$\theta_A = \theta_{max}$
  $X = L$ $y = 0$ .$\theta_A = \theta_{max}$




**Q.1(a) (iv) A cantilever of span L carries point load W at L/2 from fixed end. State value of slope at free end. [2]**

**Ans.:** A cantilever of span L carries point load 'w' at $\frac{L}{2}$ from fixed end. State value of slope and free end

**Given :**




$L_1 = \frac{L}{2}$ and $(L - L_1) = \frac{L}{2}$

From the standard case,

$\theta_B$ = Slop at free end = $\frac{WL_1^2}{2EI}$ rad. [1 mark]

$$= \frac{W\left(\frac{1}{2}\right)^2}{2\,EJ}$$

$$\theta_B = \frac{WL^2}{8EI}$$ [1 mark]

**Q.1(a) (v) State the two situations where Macaulay's method is used for finding slope and deflection of beam. [2]**

**Ans.:** Macaulay's method is used for finding slope and deflection of beam as follows.

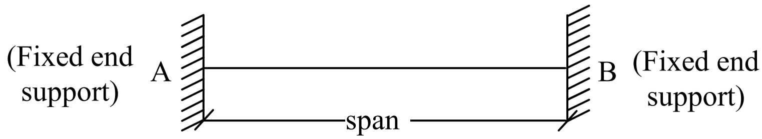
(a) Use of Macaulay's method is very convenient for cases of discontinuous and or discrete loading. [1 mark]

(b) Typically udl and vdl over the span and number of concentrated loads are conveniently handle using this technique. [1 mark]

**Q.1(a) (vi) Define "fixing" and "fixed beam". [2]**

**Ans.:** **Fixing :** A support is restrained against rotation and vertical movement is know as fixing. [1 mark]

**Fixed beam :** A beam whose end is firmly built in the support like wall, pillar or any other structure, then such beam is called fixed beam. [1 mark]




**OR**

A support at which the end slopes are zero is known as fixed beam. [1 mark]

**Q.1(a) (vii) At a continuity, adjoining spans have their distribution factors as 0.43 and 0.57. What is the meaning of these values? [2]**

**Ans.:** The values given in question are the distribution factors using these factors the unbalance moment is distributed among the two span by using this distribution factor.

Distributed moment = D.F $\times$ Unbalanced moment

**Q.1(a) (viii) Define perfect and imperfect frame. [2]**

**Ans.:** **(i) Perfect frame:** A frame made up of just sufficient numbers so that it can remain in stable equilibrium, when loaded at joints.

The condition of frame to be stable is **:** $n = 2j - 3$ [1 mark]

Where, n = numbers of members

j = number of joints

**(ii) Imperfect frame :** A frame made up of either more than or less than, just sufficient number of members to keep it in static equilibrium is called imperfect frame.

Where, $n < 2j - 3$ ………. Deficient [1 mark]

$n > 2j - 3$ ……….. Redundant frame

**Q.1(b) Attempt any TWO of the following : [8]**

**Q.1(b) (i) A pillar is square in section and has side 1 m. Values of axial and bending stress are 300 kN/m$^2$ and 287 kN/m$^2$ respectively. Determine resultant stresses. Draw resultant stress distribution diagram. Also state whether the load line is within the core or not. [4]**

**Ans.: Given :**

$\delta_t$ = 287 kN/m$^2$

$\delta_d$ = 300 kN/m$^2$

To determine resultant stresses.

$\therefore$ $\delta_{max} = \delta_d + \delta_b$

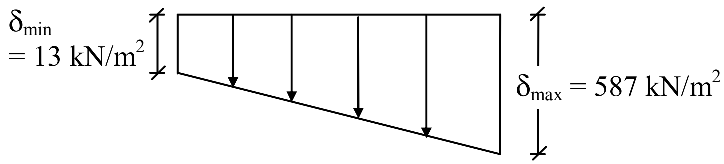
$= 300 + 287$

$= 587$ kN/m$^2$ [1 mark]

$\delta_{min} = \delta d - \delta b$

$= 300 - 287$

$= 13$ kN/m$^2$ [1 mark]




[1 mark]

Resultant stress distribution diagram

It state that load is within core because from resultant stresses it is seen that it totally compressive in nature, so that load is within core of section. [1 mark]

**Q.1(b) (ii) A short column of external diameter 250 mm and internal diameter 200 mm carries an eccentric load. Find the eccentricity for no tension condition. [4]**

**Ans.:** Given:

D = 250 mm

d = 200 mm

Solution: $\sigma min = \sigma d - \sigma b$

$$= P/A - \frac{Pe\,y}{1}$$

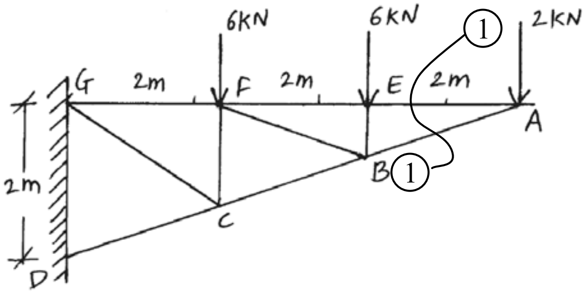
$$= \frac{P}{\dfrac{\pi(D^2 - d^2)}{4}} - \frac{P_e}{\dfrac{\pi(D^2 - d^2)}{64}} \times D/2 \qquad \ldots (1) \qquad [2 \text{ mark}]$$

For No Tension $\sigma min = 0$ put in equation (1)

$$e = \frac{(D^2 - d^2)}{8\,D} = \frac{(250^2 - 200^2)}{8 \times 25}$$

e = 51.25 mm [2 mark]

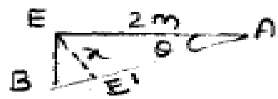
**Q.1(b) (iii)Determine the forces in the members FE, FB and CB using method of section for the truss shown in Figure.** **[4]**




**Ans.:** Take section (1) − (1) such that it passes through members whose force are required

$\tan\theta = \dfrac{2m}{6m}$

$\theta = 18.43°$




For equilibrium $\sum MB = 0$

$2 \times 2 - AE \times 0.66$

$AE = 6.0$ kN Tensile

$\sum ME = 0$

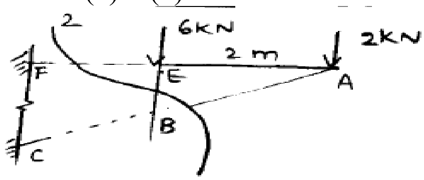
$2 \times 2 + FAB \times 0.632$

$\sin\theta = \dfrac{EE'}{2}$

$EE' = 0.632$

$AB = -\,6.32$ kN (Compressive) [1 mark]

Section (2) − (2)




Assuming EE and BE tensile

$\sum MA = 0$

$-\,6 \times 2 - BE \times 2 = 0$

$\sin\theta = \dfrac{h}{4}$

$h = 1.26$

$\therefore\ EB = -\,6$ kN (Compressive)

$\sum MB = 0$

$-\,FE \times 0.66 + 2 \times 2$

$FE = 6.00$ kN (Tensile) [1 mark]

section (3) − (3)

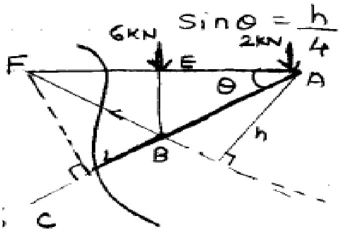
Assuming FB and BC are tensile

$\sum MA = 0$

h = 1.26

$FB \times 1.26 - 6 \times 2$

FB = 9.52 kN (Tensile) [1 mark]




$\sum MF = 0$

$6 \times 2 + 2 \times 4 + BC \times 1.26$

BC = − 15.87 kN (Compressive) [1 mark]

| | Member | Force | Nature |
|---|---|---|---|
| (i) | FE | 6.00KN | Tensile |
| (ii) | FB | 9.62KN | Tensile |
| (iii) | CB | − 15.87KN | Compressive |

**Q.2 Attempt any FOUR of the following : [16]**

**Q.2(a) A chimney having external diameter 5 m and 50 m high. It is subjected to horizontal wind pressure of 7 KPa normal to the chimney. Find the maximum bending stresses in the chimney. (C = 0.7) [4]**

**Ans.: Given**

D = 5m H = 50m $\rho = 7$ Kpa $- 7 \times 10^3$ N/m$^2$ C = 0.7

Solution

Maximum Bending stress, $\sigma b = \dfrac{M}{1} \times y$

(1) Moment of 'P' about base $= P\left(\dfrac{H}{2}\right)$ [1 mark]

(2) Total wind force P $= C \times \rho \times$ Projected area [1 mark]

$$= \frac{0.7 \times 7000(5 \times 50)\left(\dfrac{50}{2}\right) \times \dfrac{5}{2}}{\dfrac{\pi(5^4)}{64}}$$ [1 mark]

$\sigma_{max} = 2.4950$ N/m$^2$ [1 mark]

(3) $I = \dfrac{\pi(D^4)}{64}$

**Q.2(b) A hollow circular column having external diameter 2 m, carries load of 460 kN at an eccentricity of 0.8 m. Draw resultant stress diagram. (For this column Area = 2.356 $m^2$ and $I_{xx} = I_{yy} = 0.7363\ m^4$)** **[4]**

**Ans.: Given**

D = 2m, P = 460 KN, e = 0.8m, A = 2.356 $m^2$

$I \times x = I_{yy} = 0.7363\ m^4$

To find regulation stress i.e. $\delta_{max}$ and $\delta_{min}$

Direct stress = $\dfrac{P}{A} = \dfrac{460}{2.356}$ [1 mark]

$\delta_0 = 195.25\ KN/m^2$

Bending stress $(\delta_t) = \dfrac{M}{Z} = \dfrac{P.C}{\dfrac{I}{y_{max}}}$

$= \dfrac{460 \times 0.8}{\dfrac{0.7363}{1}}$

$\delta_t = 499.80\ KN/m^2$ [1 mark]

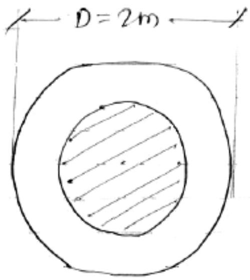
$\delta_{max} = \delta_0 + \delta_b$

$= 195.25 + 499.80$

$\delta_{max} = 695.05\ KN/m^2$ (compressive) [½ mark]

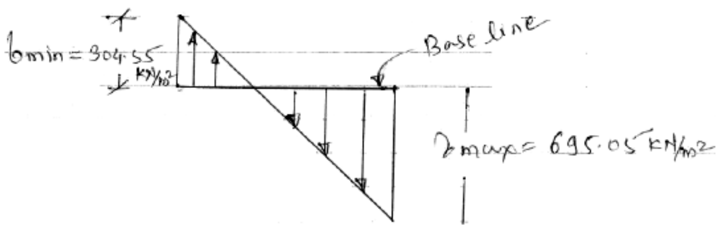
$\delta_{min} = 195.25 - 499.80$

$= -304.55$ (tensile) [½ mark]




[1 mark]

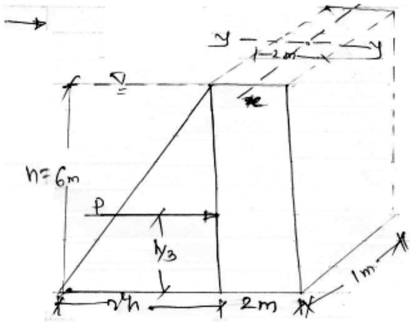
Cross section.




Resultant stress distribution dia.

**Q.2(c) A retaining wall 6 m high has uniform thickness 2 m. It retains water upto top. Determine total water pressure and net stresses at base. Draw stress diagram. Use unit wt. of water 10 kN/m$^3$ and unit wt. of wall motored is 18 kN/m$^3$.** **[4]**

**Ans.:**




Given $\nu = 10\ \text{KN/m}^3$

$\delta = 18\ \text{KN/m}^3$

Assume unit length for calculation.

(i) Total hydrostatic pressure

$$= \frac{1}{2}\nu h^2 \qquad = \frac{1}{2}\times 10\times 6^2$$ [1 mark]

$P = 180$ KN

(ii) Direct stress $\delta_0 = \dfrac{\text{Total}\ \omega t}{\text{c / s Area}}$ [1 mark]

$$= \frac{A\times h\times\delta}{A}$$

$$= h\times\delta$$

$$= 6\times 18$$

$$\delta_0 = 108\ \text{KN/m}^2$$

**OR**

$$\text{Total}\ \omega t = \delta\times V$$

$$= \delta\times A\times h$$

$$= 18\times 2\times 1\times 6$$

$$= 216\ \text{KN}$$

$$\delta_0 = \frac{\text{Total}\ \omega t}{\text{c / s Area}}$$

$$= \frac{216}{2\times 1} = 108$$

(iii) Bending stress $\delta_f = \dfrac{M}{2}$ [1 mark]

$$= \frac{P\times\frac{h}{3}}{z}$$

$$z = \frac{bd^2}{6} = \frac{1 \times 2^2}{6} = 0.667 \text{ m}^3$$

$$\delta_b = \frac{180 \times 6/3}{0.667} = 539.73 \text{ KN/m}^2$$

(iv) Resultant stresses at the base

$$\delta_{max} = \delta_0 + \delta_b = 108 + 539.73$$

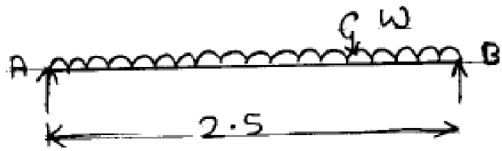
$$\delta_{max} = 647.73 \text{ KN/m}^2 \text{ (Compressive)}$$ [½ mark]

$$\delta_{min} = \delta_0 - \delta b = 108 - 539.73$$

$$= -431.73 \text{ (tensile)}$$ [½ mark]

**Q.2(d) A beam of span 2.5 m is simply supported and carries UDL w/unit length, if slope at the end is not to exceed 1.5°. Find the maximum deflection.** **[4]**

**Ans.:**




**Data:**

Slope = 1.5°

Maximum deflection = y = ?

Where,

$$\frac{dy}{dx} = \text{Slope} = 1.5°$$

$$1.5° = \frac{WL^3}{24\,EI}$$

$$1.5 \times \frac{\pi}{180} = \frac{w \times 2.5^3}{2.4\,EI}$$

$$W = 0.040\,EI$$ [2 marks]

but $$y_{max} = \frac{-SWL^4}{384\,EI}$$

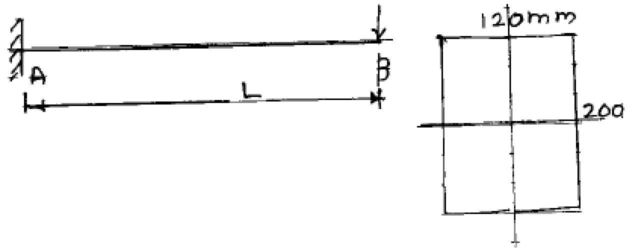
$$y_{max} = \frac{-5 \times 0.040\,EI \times 2.54}{384\,EI}$$

$$= 0.02045 \text{ m}$$

$\therefore$ $y_{max}$ = 20.45 mm downward [2 marks]

**Q.2(e) A cantilever beam has cross section 120 mm wide and 200 mm deep. If load of 6 kN acting at the free end, calculate the span of beam if slope at free end of beam is $1.5 \times 10^{-3}$ radians. Take E = 100 kN/mm$^2$. [4]**

**Ans.:**




**Data :**

W = 6 KN

$\frac{dy}{d\times B}$ = Slope = $1.5 \times 10^{-3}$ radians

E = 100 KN/mm$^2$

= $1\times 10^8$ KN/m$^2$ [1 mark]

Where,

$$I = \frac{120\times200^3}{12} = 80 \times 106 \text{ mm}^4$$ [1 mark]

$$= 80 \times 10^{-6} \text{ m}^4$$

$$\text{Slope} = \frac{dy}{d\times B} = \frac{WL^2}{2EI}$$ [1 mark]

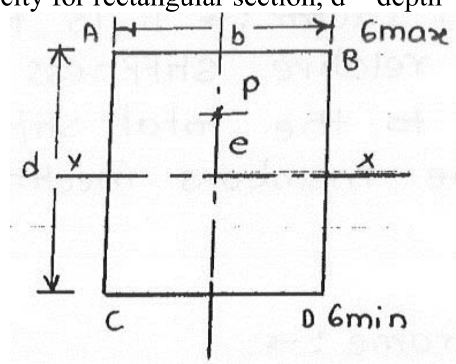
$$1.5 \times 10^{-3} = \frac{6\times L^2}{2\times1\times10^8\times80\times10^{-6}}$$

$$L^2 = 4$$

$$\therefore \quad L = 2\text{m}$$ [1 mark]

**Q.2(f) Calculate limit of eccentricity for rectangular section having width 'b' and depth 'd' and show it on sketch. [4]**

**Ans.:** Limit of eccentricity for rectangular section, d = depth



6max = 6o + 6b … [1 mark]

6min = 6o − 6b …

$$= \frac{P}{A} - \frac{M}{I_x} \times y$$

Where $M = p \times e$ [1 mark]

$$I_x = \frac{bd^3}{12}$$

$$y = \frac{d}{2}$$

$\therefore$ 6min = $\frac{P}{A} - \frac{Pe}{\left(bd^3/12\right)} \times \frac{d}{2}$

$\therefore$ For no tension 6min = 0 [1 mark]

$$0 = \frac{P}{A} - \frac{Pe}{bd^2} \times 6$$

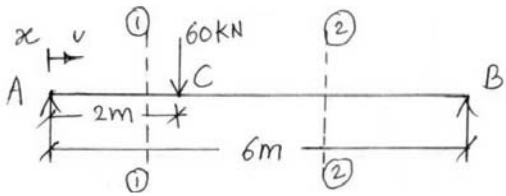
$$\frac{Pe}{bd^2} \times 6 = \frac{P}{A}$$

$\therefore$ $e \le \frac{d}{6}$ Limit of eccentricity for rectangular sec. [1 mark]

**Q.3 Attempt any FOUR of the following : [16]**

**Q.3(a) A simply supported beam of span 6m, carries a point load of 60KN at 2m from left hand support. Calculate deflection under the point load. Use Macaulay's method [4]**

**Ans.:**




(i) To find support reactions

$\sum M_A = 0 \, (-\curvearrowright +)$

$-R_B \times 6 + 60 \times 2 = 0$

$R_B = 20$ kN [1 mark]

$R_A = 60 - 20 = 40$ kN

(ii) To find slope and deflection

$EI\frac{d^2y}{dx^2} = M$ − Differential equation

$$EI\frac{d^2y}{dx^2} = 40 \times x \;\Big|_{1}\; -60(x-2) \;\Big|_{2}$$ … Moment of equation

x = 2 m   x = 6 m

Integrating w.r. to x

$$EI\frac{dy}{dx} = \frac{40x^2}{2} + C_1 \;\Big|\; \frac{-60(x-2)^2}{2} \;\Big| \qquad \ldots \text{ Slope equation (I)}$$

Again Integrating w.r. to x

$$EI \cdot y = \frac{20x^3}{3} + C_1x + C_2 \;\Big|\; \frac{-30(x-2)^3}{3} \;\Big| \qquad \ldots \text{ Deflection equation (II)}$$

[1 mark]

(iii) To calculate constants of Integration

Boundary Conditions

At x = 0, y = 0 Putting in Deflection equation (I)

$$EI\,(0) = \frac{20(0)^3}{3} + C_1\,(0) + C_2 \;\Big|\; \frac{-30}{3}\,(0 \cancel{-} 2)^3$$

$$C_2 = 0$$

At x = 6m, y = 0 Putting in Deflection equation

$$EI\,(0) = \frac{20(6)^3}{3} + C_1\,(6) + C_2 \;\Big|\; \frac{-30}{3}\,(6-2)^3$$

$$0 = 800 + 6C_1$$

$$C_1 = -\,133.33$$

[1 mark]

Putting values of $C_1$ and $C_2$ in slope and deflection equation and rewriting equation.

$$EI.\frac{dy}{dx} = 20x^2 - 133.33 \;\Big|\; -30(x-2)^2 \;\Big| \qquad \ldots \text{ Final slope equation}$$

$$EI.y = \frac{20}{3}x^3 - 133.33 \;\Big|\; \frac{-30}{3}\,(x-2)^3 \qquad \ldots \text{ Final deflection equation}$$

(iv) Deflection under point load

At x = 2m, $y = y_C$ Putting in final deflection equation

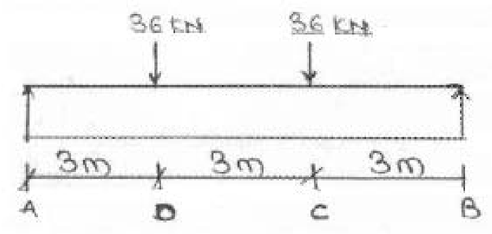
$$EI.y_C = \frac{20}{3}\,(2)^3 - 133.33\,(2) - 0$$

$$y_C = -\frac{213.326}{EI} \qquad \ldots \text{ Deflection below point load}$$

[1 mark]

**Q.3(b) A simply supported beam of span 9 m carries two point loads of equal magnitude 36 kN at 3 m from both ends. Calculate values of integration constants and write Macaulay's slope and deflection equation.** **[4]**

**Ans.:** From the given data a simply supported beam is drawn as shown in figure.




**Step 1 :** Given

$W_1 = 36$ KN  $W_2 = 36$ KN  $L = 92$

**Step 2 :** To Find support Reaction $R_A$ and $R_B$

$+\uparrow \; -\downarrow \quad \Sigma F_y = R_A - 36 - 36 + R_B$

$\therefore \quad R_A + R_B = 72 \quad \ldots (1)$

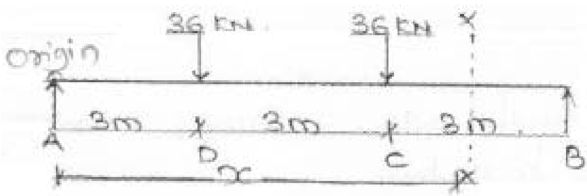
$$\Sigma MA = 36 \times 3 + 36 \times 6 - 9 \times R_B$$
$$= 108 + 216 - 9R_B$$
$$R_B = 36 \text{ KN}$$

Substituting the value of $R_B$ in equation (1) we get

$R_A = 36$ KN OR Due to symmetrical loading $R_A = R_B = 36$ KN

**Step 3 :** To construct slope and deflection equation by Macaulay's Method.

Consider section x – x at a distance x from A in portion BC as shown in figure below.




$\therefore$ The general bending moment equation at a distance x from A, by sign convention

+ −

$$EI\frac{d^2y}{dx^2} = Nx = |36x| - |36(x-3)| - |36(x-6)| \quad \ldots (1) \text{ [½ mark]}$$

Integrating equation (1) w.r.t. x

$$EI\frac{dy}{dx} = \left|\frac{36x^2}{2} + C_1\right| - \left|\frac{36(x-3)^2}{2}\right| - \left|\frac{36(x-6)^2}{2}\right| \quad \ldots (2) \quad \text{[½ mark]}$$

Integrating equation (2) w.r.t. x

$$EI_y = \left|\frac{36}{2}\frac{(x)^3}{3} + C_1x + C_2\right| - \left|\frac{36}{2}\frac{(x-3)^3}{3}\right| - \left|\frac{36}{2}\frac{(x-6)^3}{3}\right| \quad \ldots (3) \qquad [½ \text{ mark}]$$

**Step 4 :** Applying boundary condition to find values of constants $C_1$ and $C_2$.

At simple support A  x = 0,  deflection y = 0

and  At simple support B  x = 9,  deflection y = 0.  [½ mark]

by conditions x = 0, y = 0

Substituting these values in equation (3) upto first bracket. We get

$0 = 0 + 0 + C_2$.

C2 = 0

by condition 2.  x = 9, y = 0  [½ mark]

Substituting these values in equation 3 upto all bracket.

$$0 = \left|36\times\frac{(9)^3}{6} + 9C_1 + 0\right| - \left|36\frac{(9-3)^3}{6}\right| - \left|36\frac{(9-6)^3}{6}\right| \qquad [1 \text{ mark}]$$

$0 = 4374 + 9C_1 - 1296 - 162$

$\therefore\ 9C_1 = -2916$

$$\therefore\quad C_1 = \frac{-2916}{9}$$

$C_1 = -324$  [½ mark]

Hence $C_1 = -324$ and $C_2 = 0$

**Q.3(c) State advantages and disadvantages of fixed beams** **[4]**

**Ans.: Advantages of fixed Beam** [Any 2 - 2 marks]

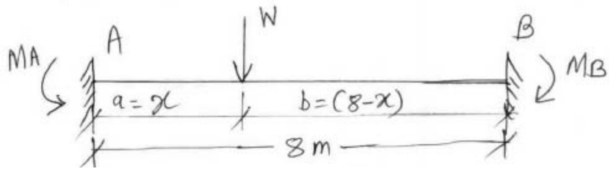
- The slopes at the end support in case of fixed beam are zero but in case of simply supported beam slopes are maximum.
- The maximum bending moment and deflection are less as composed to simply supported beam for same span and loading.
- Fixed beam is more strong, stable and stiff than simply supported beam.
- The cross section required for fixed beam is smaller and steel required is less due to lesser B.M. and hence it is economical as composed to simply supported beam.

**Disadvantages of fixed beam** [Any 2 - 2 marks]

- If any one of the support sinks to a small extent, it induces additional moment at each end.
- since both the end of beam are fixed, temperature stresses are developed due to variation in temperature.
- Complete fixity cannot be achieved.

**Q.3(d) A fixed beam of span 8m carries a point load W at distance 'x' from left hand support. If the moment at the left end is twice the moment at right end evaluate 'x'. [4]**

**Ans.:** A fixed beam of span 8m carries a point load W at a distance 'x' from the left hand support. If the moment at the left end is twice that at the right end evaluate 'x'.




$M_A = 2\,M_B \qquad \ldots (1)$ [1 mark]

To find fixed end moments

$$M_A = -\frac{wab^2}{L^2} = \frac{-w(x)(8-x)^2}{8^2}$$ [½ mark]

$$M_B = -\frac{wa^2b}{L^2} = \frac{-w\left(x^2\right)(8-x)}{8^2}$$ [½ mark]

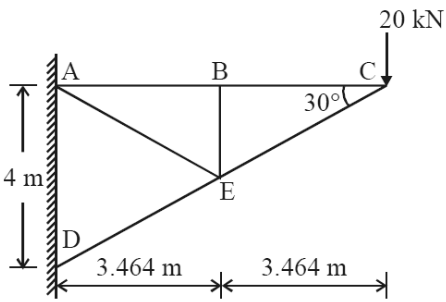
Putting values of $M_A$ and $M_B$ in equation (1)

$\therefore \quad M_A = 2\,M_B$

$$\frac{w(x)(8-x)^2}{8^2} = 2\left[\frac{w\left(x^2\right)(8-x)}{8^2}\right]$$ [1 mark]

$$(8-x) = 2x$$
$$3x = 8$$
$$x = \frac{8}{3}$$
$$x = 2.67\text{m}$$ [1 mark]

**Q.3(e) For the truss shown in fig. below, determine nature and magnitude of forces in members BC, CE, AE and DE. Use method of joints. [4]**

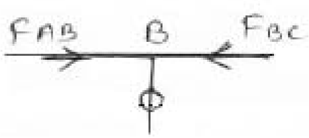


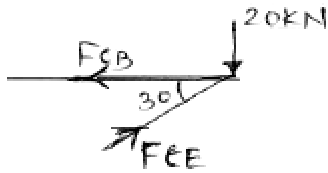

**Ans.:** To find distance BE

$$\tan 30 = \frac{BE}{BC}$$

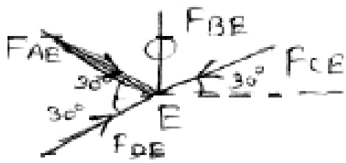
$\therefore$ BE = BC tan 30
= 3.464 tan 30 = 1.999 m $\cong$ 2m

Consider joint C
FBD for joint C
$\therefore$ $\sum fy = 0$
$+ 20 - F_{CE} \sin 30 = 0$
$\therefore$ $- E_{CE} \sin 30 = - 20$
$\therefore$ $F_{CE} = + 40$ KN (compressive) [1 mark]
$\sum fx = 0$
$F_{CB} + F_{CE} \cos 30 = 0$
$\therefore F_{CB} = 34.64$ KN [1 mark]
Consider joint B
$\sum fx = 0$
$F_{AB} - F_{BC} = 0$
$\therefore$ $F_{AB} = F_{BC} = 34.64$ KN
$\sum fy = 0$
$\therefore$ $F_{BE} = 0$ [1 mark]



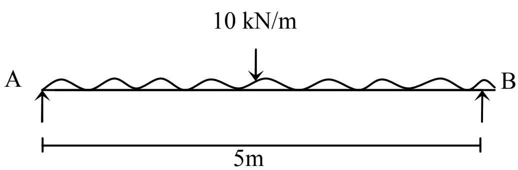




Consider joint E
$- 34.64 - F_{AE} \cos30 - F_{DE} \cos 30 = 0$
$\therefore$ $F_{AE} + F_{DE} \cos 30 = - 34.64$ … (i)
$\sum fy = 0$
$- 20 + F_{AE} \sin 30 - F_{DE} \sin 30 = 0$
$F_{AE} \sin 30 - F_{DE} \sin 30 = 20$ … (ii)
Solving equation (i) and (ii) simultaneously
$F_{AE} = 0$ and $F_{DE} = - 40$ KN (comparative) [1 mark]

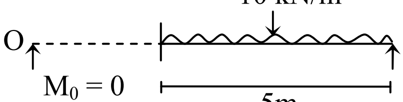



**Q.3(f) A propped cantilever AB of span 5m carries u.d.l of 10Kn/m over entire span. A is fixed and B is simply supported using three moment theorem find support moment and draw B.M.D** **[4]**

**Ans.:**

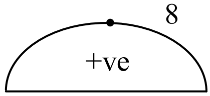


For using 3 moment theorem

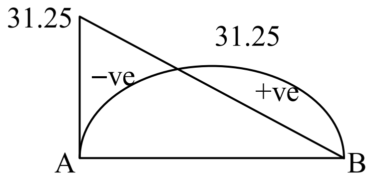



$M_0 = 0$
$L_0 = 0$
$a_0 = 0$
$x_0 = 0$

$\dfrac{WL^2}{8} = 31.25$




Considering AB to be S.S. and Finding BMD




$\therefore \quad L_1 = 5$

$x_1 = \dfrac{2}{3} \times 5 \times 31.25 = 104.167$

$x_1 = \dfrac{5}{2} = 2.5$

Applying 3 moment theorem

$$M_0L_0 + 2\,M_A\,[L_0 + L_1] + M_B\,L_1 = -6\left[\frac{a_0x_0}{L_0} + \frac{a_1x_1}{L_1}\right]$$

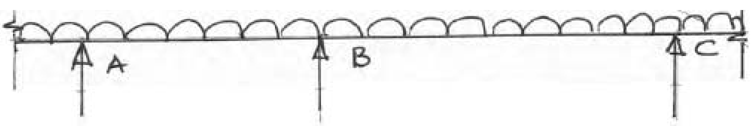
$$2M_A \times 5 = \frac{-6 \times 104.167 \times 2.5}{5}$$

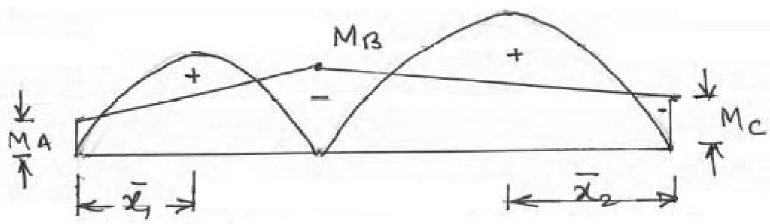
$M_A = -31.25$ kNm

**Q.4 Attempt any FOUR of the following : [16]**

**Q.4(a) State Clapeyron's theorem for a continuous beam having same moment of inertia as well as for different moment of inertia. State meaning of each term with sketch. [4]**

**Ans.:**




[½ mark]

Clapeyron's Theorem of three moments. If AB and BC are any two consecutive spans of a continuous beam having uniform moment of Inertia, supported at A, B and C and subjected to an external loading, the support moments $M_A$, $M_B$ and $M_C$ at supports A, B and C are given by the relation.

$M_A \times L_1 + 2M_B\,(L_1 + L_2) + M_C \times \ell_2 - \left(\dfrac{6a_1\bar{x}_1}{\ell_1} + \dfrac{6a_2\bar{x}_2}{\ell_2}\right)$ [2 marks]

Clapeyron's Theorem when M.I. of beam is varying

$$M_A \times \frac{L_1}{I_1} + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C \times \frac{L_2}{I_2}$$

$$= -\left(\frac{6a_1\bar{x}_1}{I_1L_1} + \frac{6a_2\bar{x}_2}{I_2L_2}\right)$$ [1 mark]

where,

$L_1$ = Length of the span AB

$I_1$ = Moment of Inertia of beam for span AB

$a_1$ = Area of free B.M. diagram for the span AB [½ mark]

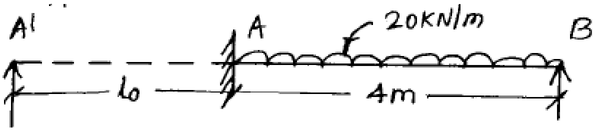
$\bar{x}_1$ = Distance of centroid of free BMD of span AB from 'A'

$\bar{x}_2$ = Distance of centroid of free BMD of span BC from 'C'

$\therefore$ Similarly $L_2$, $I_2$ and $a_2$ for the span BC.

**Q.4(b) A propped cantilever AB of span 4m is fixed at A and propped at B Carrying Udl Of 20KN/m. Calculate support moment using Clapeyron's theorem. Draw SFD and BMD.** **[4]**

**Ans.: Propped Cantilever Beam**




Consider the imaginary span AA′ at fixed support having zero span and loading. Apply Clapeyron's Theorem for span A′A and AB.

$$M_{A'}(\cancel{L_0}) + 2M_A(L_0 + L_1) + M_B(\cancel{L_1}) = -\left[\frac{6A_0\bar{X}_0}{L_0} + \frac{6A_1\bar{X}_1}{L_1}\right]$$ [1 mark]

Simply supported BMD

for span A′A = 0

span AB $\quad M_{max} = \dfrac{20 \times 4^2}{8} = 40$ kN.m

$A_0 = 0, \quad \overline{X_0} = 0, \quad A_1 = \dfrac{2}{3}bh = \dfrac{2}{3} \times 40 \times 4$

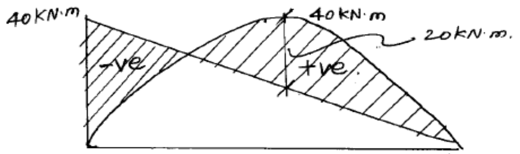
$A_1 = 106.67\ \text{kNm}^2$ [1 mark]

$\overline{X_1} = \dfrac{4}{2} = 2$ m

$M_B = 0$ For Simple support

$$2M_A\,(0 + 4) = -\left[\frac{6 \times 0 \times 0}{0} + \frac{6 \times 106.67 \times 2}{4}\right]$$

$M_A = -40$ kN.m [1 mark]

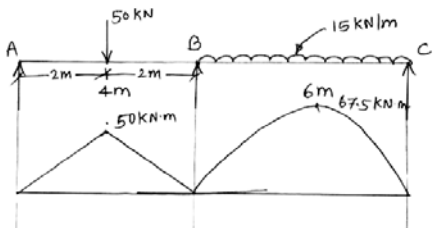



FINAL BMD

[1 mark]

**Q.4 (c) A beam ABC is simply supported at A B and C. span AB and BC are of span 4m and 6m respectively. AB carries a central point load of 50KN and BC carries a udl of 15KN/m over the entire span. Calculate support moment at B using three moment theorem.** **[4]**

**Ans.:**




S.S. BMD

Calculate S.S. Bending moments

For span AB

$$M_{max} = \frac{WL}{4} = \frac{50 \times 4}{4} = 50 \text{ kN.m}$$

For span BC

$$M_{max} = \frac{WL^2}{8} = \frac{15 \times 6^2}{8} = 67.5 \text{ kN.m}$$ [1 mark]

Applying Three moment theorem for span AB & BC

$$M_A \times L_1 + 2M_B (L_1 + L_2) + M_C (L_2) = -\left[\frac{\delta\ A_1\bar{X}_1}{L_1} + \frac{\delta\ A_2\bar{X}_2}{L_2}\right]$$ [1 mark]

$M_A = M_C = 0$ Simple supports

$$A_1 = \frac{1}{2} \times 4 \times 50 = 100 \text{ kNm}^2$$

$$A_2 = \frac{2}{3} bh = \frac{2}{3} \times 6 \times 67.5 = 270 \text{ kN.m}^2$$

$$\bar{X}_1 = \frac{4}{2} = 2 \text{ m} \qquad \bar{X}_2 = \frac{6}{2} = 3 \text{ m}$$ [1 mark]

$$0 \times (L_1) + 2M_B (6 + 4) + M_O (6) = -\left[\frac{6 \times 100 \times 2}{4} + \frac{6 \times 270 \times 3}{6}\right]$$

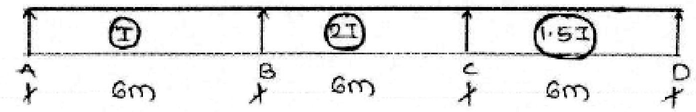
$$20\ M_B = [300 + 810]$$

$[M_B = 55.5 \text{ kN.m}]$ … Support moment

[1 mark]

**Q.4(d) Beam ABCD is simply supported at A & D and is continuous over B & C. Determine distribution factors. (AB = BC = CD = 6 m) (IAB = I, IBC = 2I, ICD = 1.5 I).** **[4]**

**Ans.:** From the given data




**Step 1 :** To calculate stiffness factor (K)

**Joint B**

$$K_{BA} = \frac{3EI}{L_1} = \frac{3EI}{6} = 0.5\ EI$$

$$K_{BC} = \frac{4EI}{L_2} = \frac{4E(2I)}{6} = 1.33\ EI$$

$$\begin{aligned}\Sigma K &= K_{BA} + K_{BC} \\ &= 0.5EI + 1.33\ EI \\ &= 1.83\ EI\end{aligned}$$

**Joint C**

$$K_{CB} = \frac{4EI}{L_2} = \frac{4E(2I)}{6} = 1.33\ EI$$

$$K_{CD} = \frac{3EI}{L_3} = \frac{3E(1.5I)}{6} = 0.75\ EI$$

$$\begin{aligned}\Sigma K &= K_{CB} + K_{CD} \\ &= 1.33\ EI + 0.75\ EI \\ &= 2.08\ EI\end{aligned}$$

**Step 2 :** Distribution factor calculation

**Joint B**

$$(Df)_{BA} = \frac{K_{BA}}{EK} = \frac{0.5EI}{1.83EI} = 0.27$$ [1 mark]

$$(Df)_{BC} = \frac{K_{AC}}{EK} = \frac{1.33EI}{1.83EI} = 0.73$$ [1 mark]

Check $(Df)_{BA} + (Df)_{BC}$

$= 0.28 + 0.72$

$= 1$

**Joint C**

$$(Df)_{CB} = \frac{K_{CB}}{EK} = \frac{1.33EI}{2.08EI} = 0.64$$ [1 mark]

$$(Df)_{CD} = \frac{K_{CD}}{EK} = \frac{0.75EI}{2.08EI} = 0.36$$ [1 mark]

Check $(Df)_{CB} + (Df)_{CD} = 0.64 + 0.36 = 1$

**OR**

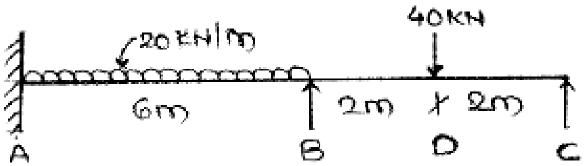
| Joint | Member | Stiffness factor | Total Stiffness | Df | |
|---|---|---|---|---|---|
| B | BA | $\frac{3EI}{L} = \frac{3EI}{6} = 0.5\text{ EI}$ | 1.83 EI | 0.27 | [1 mark] |
| | BC | $\frac{4EI}{L} = \frac{8EI}{6} = 1.33\text{ EI}$ | | 0.73 | [1 mark] |
| C | CB | $\frac{4EI}{L} = \frac{8EI}{6} = 1.33\text{ EI}$ | 2.083 EI | 0.64 | [1 mark] |
| | CD | $\frac{3EI}{L} = \frac{3E \times 1.5I}{6} = 0.75\text{ EI}$ | | 0.36 | [1 mark] |

**Q.4(e) A beam ABC is fixed at A and is supported at B and C. 20 kN/m u.d. load acts on AB and 40 kN point load acts at centre of BC. If $DF_{BA}$ = 0.57 and $DF_{BC}$ = 0.43. Determine support moment using moment distribution method. *l*(AB) = 6 m and *l* (BC) = 4 m.** **[4]**

**Ans.: Given**

$L_1$ = 6m $L_2$ = 4m W = 20 kN/m W = 40 kN

**From given data**




**Step 1 :** Fixed End Moment Calcualtion

For Span AB [1 mark]

$$M_{AB} = \frac{-WL_1^2}{12} = \frac{-20 \times (6)^2}{12} = -60\text{ kN.m}$$

$$M_{BA} = \frac{+W \cdot L_1^2}{12} = \frac{+20 \times (6)^2}{12} = +60\text{ kN.m}$$

For Span BC [1 mark]

$$M_{BC} = \frac{-WL_2}{8} = \frac{-40 \times 4}{8} = -20\text{ kN.m}$$

$$M_{CB} = \frac{+WL_2}{8} = \frac{+40 \times 4}{8} = +20\text{ kN.m}$$

**Step 2 :** Distribution Factor (Given)

$(DF_{BA})$ = 0.57 $(DF_{BC})$ = 0.43

**Step 3 :** Moment Distribution Table

| | A | B | | C |
|---|---|---|---|---|
| D.F. | | 0.57 | 0.43 | |
| FEM | − 60 | +60 | − 20 | + 20 |
| | | | | − 20 |
| Release C and carry over to B | | | −10 | |
| Initial moments | − 60 | +60 | −30 | 0 |
| $1^{st}$ distribution | − 8.55 | − 17.1 | − 12.9 | |
| Final moment | − 68.55 | + 42.9 | − 42.9 | 0 |

[2 marks]

**Step 4 :** Free Bending Moment

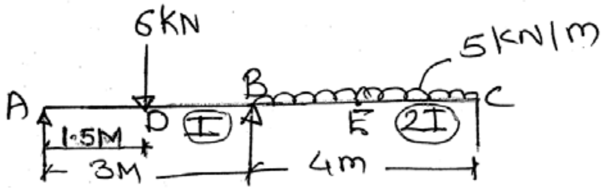
$M_A = 0$, $M_B = 0$ Maximum Bending Moment

$$= \frac{WL^2}{8} = \frac{20 \times 6^2}{8} = 90 \text{ kN.m}$$

$M_B = 0$, $M_C = 0$ Maximum Bending Moment

$$= \frac{WL}{4} = \frac{40 \times 4}{4} = 40 \text{ kN.m}$$

**Q.4(f) Calculate support moments for given continuous beam by moment distribution method.** **[4]**




**Ans.: Step 1**

For Span AB

$$M_{ab} = -\frac{WL}{8} = -\frac{6 \times 3}{8} = -2.25 \text{ kN.m}$$

$$M_{ba} = +\frac{WL}{8} = +\frac{6 \times 3}{8} = +2.25 \text{ kN.m}$$

For Span BC

$$M_{bc} = -\frac{WL^2}{12} = -\frac{5 \times 4^2}{12} = -6.67 \text{ kN.m}$$

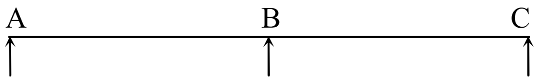
$$M_{cb} = +\frac{WL^2}{12} = +\frac{5 \times 4^2}{12} = +6.67 \text{ kN.m}$$

[1 mark]

**Step 2 :** Determination of distribution factor

| Joint | Member | Relative Stiffness | Total Stiffness | Distribution factor |
|---|---|---|---|---|
| B | BA | $\frac{3EI}{L} = \frac{3EI}{3}$ $= 1\ EI$ | 2.5 EI | 0.40 |
| | BC | $\frac{3EI}{L} = \frac{3 \times E \times 2I}{4}$ $= 1.5\ EI$ | | 0.60 |

[1 mark]




| A | B | | C | Joint |
|---|---|---|---|---|
| AB | BA | BC | CB | Member |
| | 0.40 | 0.60 | | Distribution factor |
| −2.25 | +2.25 | −6.67 | +6.67 | Fixed end moment |
| +2.25 | +1.125 | −3.335 | −6.67 | Release A and C carry forward to B |
| 0 | +3.375 | −10.005 | 0 | Initial moment |
| | +2.652 | +3.978 | | Distribute@B |
| 0 | +6.027 | −6.027 | 0 | Final moment |
| $M_B = M_C = 0$ | | $M_B = 6.027$ kN.m (Hogging) | | |

[2 marks]

**Q.5 Attempt any TWO of the following : [16]**

**Q.5(a) A circular chimney has external diameter 60% more than internal diameter. The height of chimney is 32 m and is subjected to a horizontal wind pressure of 1.75 kN/m$^2$. Find out the diameter of chimney so as to avoid tension at the baseof chimney and also draw stress distribution diagram unit wt of chimney material is 18 kN/m$^3$ and c = 0.60. [8]**

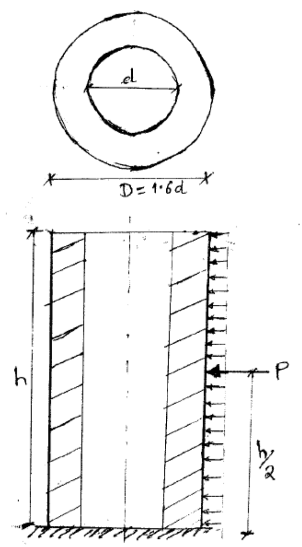
**Ans.:** Where h $= 32$ m $\quad \delta_m = 18$ KN/m$^3$

C $= 0.6$

pd $= 1.75$ KN/m$^2$

$f_{max} = ?$

$f_{min} = ?$




No tension at base

Direct stress due to self-height (fd) :

$$f_d = \frac{\delta_m A h}{A} = 18 \times 32 = 576 \text{ kN/m}^2$$ [1 mark]

Sending Stress due to kind load ($f_b$) :

Bending moment due to mind force (M)

$M = P \times h/2$

$$M = (0.6 \times 1.75)\,(D \times 32)\left(\frac{32}{2}\right)$$ [2 mark]

$M = 860.16d$ KN.m .... $D = 1.6d$

Moment of Inertia (1) about bending axis

$$I = \frac{\pi\left(D^4 - d^4\right)}{64} = \frac{\pi\left[(1.6d)^4 - d^4\right]}{64} \qquad \text{[1 mark]}$$

$I = 0.273d^4 \quad m^4$

and Distance of extreme fibre from N.A

$$y = \frac{D}{2} = \frac{1.6d}{2} = 0.8D$$

$$\therefore \; f_b = \frac{M}{I}\cdot y = \frac{860.16d}{0.273d^4}\times 0.8d \qquad \text{[1 mark]}$$

$$\therefore \; f_b = \frac{2520.62}{d^2} \; KN/m^2 \qquad \text{[1 mark]}$$

To avoid tension at base of chimney

$f_d = f_b$

$$\therefore \; 576 = \frac{2520.62}{d^2}$$

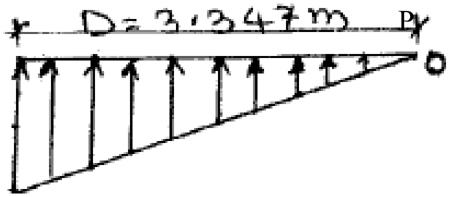
$\therefore$ Internal diameter = d = 2.92m and External diameter = D = 3.347m [1 mark]

Extreme fibre stress at the base of chimney

$f_d + f_b = f_{max}$

$2(576) = 1152$ KN/m$^2$

ω the leeward side.



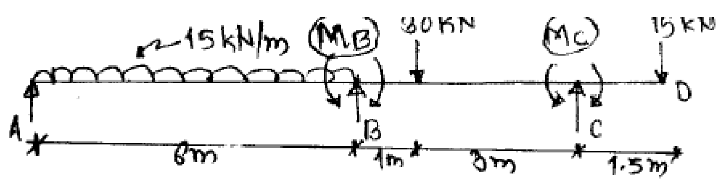

$f_{min} = fd - fb$

$f_{min} = 0$ KN/m$^2$

at the wind word side

**Q.5(b) A beam ABCD is simply supported at A,B,C and CD is overhang. AB = 6 m BC = 4 m and CD = 1.5 m. Span AB carried udl of 15KN/m over entire span and BC carries point load of 30 KN at 1 m from support B and a point load of 15 KN acts at free end. Determine support moments using moment distribution method and draw BMD. [8]**

**Ans.:**



Distribution factors at it B −DF

It B { BA relative stiffness = $\frac{3EI}{6}$ 0.4

BC = $\frac{3EI}{4}$ 0.6 }

Total stiffness = 1.25 EI $\sum = 70$ [1 mark]

Fixed end moment (FEM)

$M_{AB} = \frac{15\times(6)^2}{12} = -45$ KN.m (anticlockwise) [½ mark]

$M_{BA} = \frac{15\times(6)^2}{12} = +45$ KN.m (clockwise) [½ mark]

$M_{BC} = \frac{30\times1\times(3)^2}{(4)^2} = -16.875$ KN.m (anticlockwise) [½ mark]

$M_{CB} = \frac{30\times(1)^2\times3}{(4)^2} = +7.5$ KN.m (Clockwise) [½ mark]

$M_{CD} = 15 \times 1.5 = -22.5$ KN.m (anticlockwise) [½ mark]

Sagging BM for (S/S for every span)

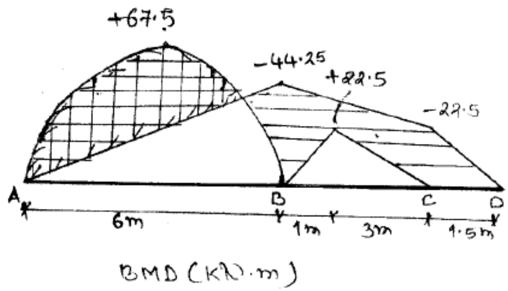
$\overset{+}{M}_{AB} = \frac{15\times(6)^2}{8} = +67.5$ KN.m [½ mark]

$\overset{+}{M}_{BC} = \frac{30\times9\times3}{4} = +22.5$ KN.m

Moment distribution table [2 mark]

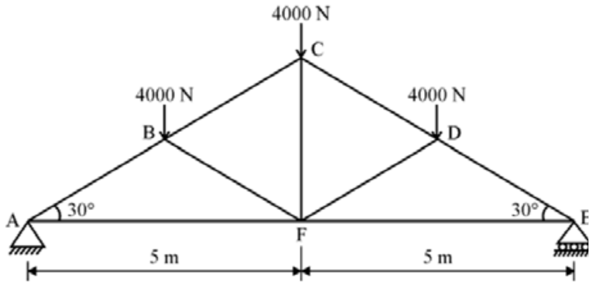
| Joint | A | B | B | C | C | D |
|---|---|---|---|---|---|---|
| D.F. | – | 0.4 | 0.6 | 1 | 0 | – |
| FEM<br>Release A &<br>Balance C | −45<br>+45 | +45<br><br>+22.5 | −16.875<br><br>+7.50 | +7.5<br>+15.0 | −22.5 | – |
| Initial Moment<br>Distribute 'B' | 0 | + 67.50<br>− 23.25 | − 9.375<br>− 34.875 | +22.5 | − 22.5 | – |
| Final moment | 0 | +44.25 | −44.25 | +22.5 | −22.5 | – |

Release 'A' = Because of S/S end Balance 'C' = Beause of overhang

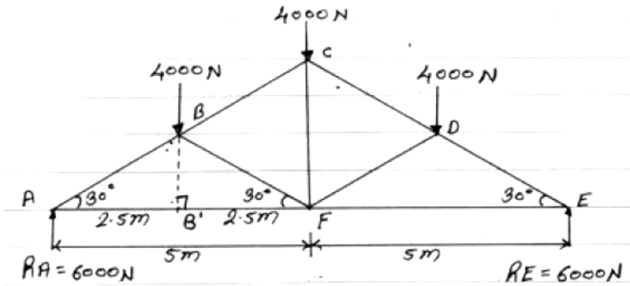



[2 marks]

**Q.5(c) Determine the nature and magnitude of forces in the members (AB, BC, FD & CF) of frame as shown in fig. Also find support reaction using method of joints.** **[8]**




**Ans.:**




(i) Support reactions

Due to symmetrical loading

$$R_A = R_E = \frac{\text{Total load}}{2} = \frac{4000 \times 3}{2}$$

$\therefore \quad R_A = R_E = 6000$ N [1 mark]

(ii) Geometrical Properties

Length of member CF

CF = AF $\times$ tan 30 = 5 $\times$ tan 30 = 2.88m

Length of member AC

$$AC = \sqrt{AF^2 + CF^2} = \sqrt{5^2 + 2.88^2} = 5.77\text{m}$$

$$\therefore \quad AB = BC = \frac{AC}{2} = \frac{5.77}{2} = 2.88\text{m}$$

Length of perpendicular BB'

BB' = AB $\times$ sin30 = 2.88 $\times$ sin30

BB' = 1.44m

Length of AB' = AB·cos30

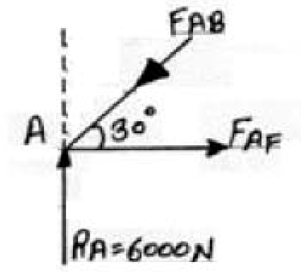
= 2.88·cos30

AB' = 2.50m

$\therefore \quad \angle BFA = \angle BAF = 30°$

Assume the directions of forces as shown in FBD

(iii) Consider joint A



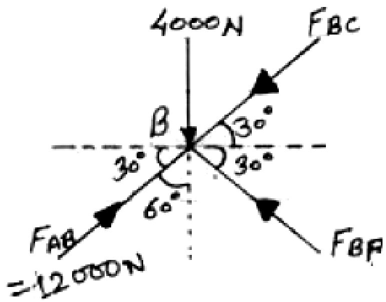

$\sum Fy = 0$

$\therefore \quad 6000 - F_{AB} \sin 30 = 0$

$\therefore \quad F_{AB} = \dfrac{6000}{\sin 30} = 12000$ N (Compressive) [2 mark]

− ve sign indicate force is compressive in member

(iv) Consider joint B




$\sum Fx = 0$

$F_{AB} \cos 30 - F_{BC} \cos 30 - F_{BF} \cos 30 = 0$

$\therefore \quad 1200 \cos 30 - 0.866\ F_{BC} - 0.866\ F_{BF} = 0$

$F_{BC} + F_{BF} = 12000$ N … (i)

$\sum Fy = 0$

$-4000 + F_{AB} \sin 30 + F_{BF} \sin 30 - F_{BC} \sin 30 = 0$

$-4000 + 12000 \cdot \sin 30 + 0.5\ F_{BF} - 0.50\ F_{BC} = 0$

$-0.50\ F_{BC} + 0.5\ F_{BF} = -2000$ ….

$\therefore \quad F_{BC} - F_{BF} = 4000$ … (ii)

Solving equation (i) and equation (ii) we get

$F_{BC} = 8000$ N (Compressive) [2 marks]

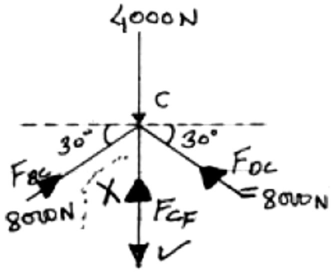
$F_{BF} = 4000$ N (Compressive)

From symmetry the force in member $F_D$

$F_{FD} = F_{BF} = 4000$N (Compressive) [1 mark]

Assume the direction of the forces as shown $F_{BD}$

(v) Consider joint C




$\therefore$ $F_{BC} = F_{DC} = 8000$ N

$\therefore$ $\sum F_y = 0$

$-4000 + F_{BC} \sin 30 + F_{DC} \sin 30 + F_{CF} = 0$

$-4000 + 8000 \cdot \sin 30 + 8000 \cdot \sin 30 + F_{CF} = 0$

$\therefore$ $F_{CF} = -4000$ N

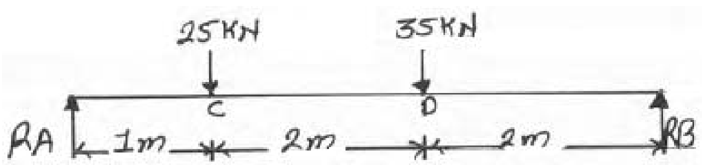
$-$ sign indicates the force in $F_{CF}$ is tensile

$\therefore$ $F_{CF} = 4000$ N (Tensile) [2 marks]

**Q.6 Attempt any TWO of the following : [16]**

**Q.6(a) A simply supported beam is subjected to two point loads 25 KN and 35 KN at 1 m and 3 m from the left support respectively. Span of the beam is 5 m. Calculate deflection under 25 KN. Load by Macaulay's method. Take E = 2 × $10^5$ N/mm$^2$, I = 3 × $10^8$ mm$^4$. [8]**

**Ans.:**




$E = 2 \times 10^5$ N/mm$^2$ $= 2 \times 10^8$ MN/m$^2$

$I = 3 \times 10^8$ mm$^4$ $= 3 \times 10^{-4}$ m$^4$

Support Reaction, $\sum Fy = 0$ $R_A + R_B = 60$ KN

$\sum M_A = 0$

$(25 \times 1) + (35 \times 3) - 5R_B = 0$

$\therefore$ $R_B = 26$ KN $\therefore$ $R_A = 34$ KN [1 mark]

Consider a section x–x at a distance x from A in portion DB

$M_X = 34.x - 25(x-1) - 35(x-3)$ [1 mark]

But,

$$EI\frac{d^{2}y}{dx^2} = M_X = 34x \;\vert\; -25(x-1) \;\vert\; -35(x-3) \quad \ldots (A)$$ [½ mark]

Integrating equation A. w.r. to x

$$EI\,\frac{dy}{dx} = \frac{34x^2}{2} + C_1 - \Big|\,\frac{25(x-1)^2}{2} - \Big|\,\frac{35(x-3)^2}{2} \qquad \ldots (B) \qquad [½ \text{ mark}]$$

Integrating equation B w.r. to x.

$$EI\cdot y = \frac{34x^3}{6} + C_1x + C_2 - \Big|\,\frac{25(x-1)^3}{6} - \Big|\,\frac{35(x-3)^3}{6} \qquad \ldots (C)$$

Apply Boundary conditions to find $C_1$ and $C_2$ values

At A, $x = 0$, $y = 0$ put in equation (C).

$$0 = 0 + C_1(0) + C_2 \qquad \therefore \quad C_2 = 0 \qquad [1 \text{ mark}]$$

At, B, $x = 5\text{m}$, $y = 0$, put in equation C

$$0 = \frac{34(5)^3}{6} + C_1\cdot 5 + 0 - \frac{25(5-1)^3}{6} - \frac{35(5-3)^3}{6}$$

$$0 = 708.33 + 5C_1 - 266.67 - 46.667$$

$$\therefore \quad C_1 = \frac{-394.99}{5} = -79$$

$$C_1 = -79 \qquad [1 \text{ mark}]$$

Substitute the values of $C_1$ and $C_2$ in equation B and C

$$EI\,\frac{dy}{dx} = \frac{34x^2}{2} - 79 - \frac{25(x-1)^2}{2} - \frac{35(x-3)^2}{2} \ldots \text{ Slope equation} \qquad [1 \text{ mark}]$$

$$EI\cdot y = \frac{34x^3}{6} - 79x - \frac{25(x-1)^3}{6} - \frac{35(x-3)^3}{6} \quad \ldots \text{ Deflection equation} \qquad [1 \text{ mark}]$$

Deflection under 25KN point load,

Put $x = 1\text{m}$, in deflection equation

$$EI\cdot y = \frac{34(1)^3}{6} - 79\times 1 = -73.333$$

$$y_C = \frac{-73.333}{EI}$$

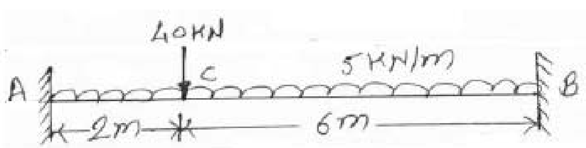
$$y_C = \frac{-73.333}{2\times 10^8 \times 3\times 10^{-4}} = 1.222\times 10^{-3}\text{m}$$

$$y_C = 1.222 \text{ mm} \qquad [1 \text{ mark}]$$

Deflection under 25 KN point load is 1.227 mm

**Q.6(b) A fixed beam of span 8 m carries 5 kN/m udl over entire length along with a point load of 40 kN at 2 m from left hand support. Find net BM at point load and draw BMD and SFD.** **[8]**

**Ans.:**



Let, a = 2m, b = 6m, L = 8m

(i) Support moments

$$M_A = -\left[\frac{wl^2}{12}+\frac{Wab^2}{L^2}\right] = -\left[\frac{5\times 8^2}{12}+\frac{40\times 2\times 6^2}{8^2}\right]$$

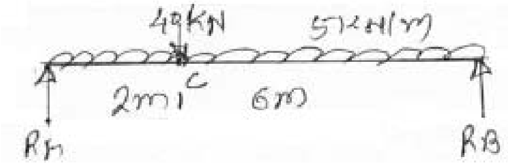
$M_A = -\ 71.67$ KN.m [1 mark]

$$M_B = -\left[\frac{WL^2}{12}+\frac{Wa^2b}{L^2}\right] = -\left[\frac{5\times 8^2}{12}+\frac{40\times 2^2\times 6}{8^2}\right]$$

$M_B = -\ 41.67$ KN.m [1mark]

(ii) Free B.M ordinate below point load.

(a) Reactions




$\sum Fy = 0$; $R_A + R_B = 80$KN

$\sum M@A = 0$;

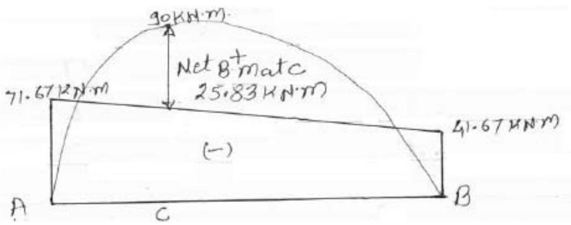
$(5 \times 8 \times 4) + (40 \times 2) - 8R_B = 0$

$\therefore\ R_B = 3$KN

$\therefore\ R_A = 50$ KN

B mat C [1 mark]

$MC = 50 \times 2 - 5 \times 2 \times 1 = 90$KN.m




[1 mark]

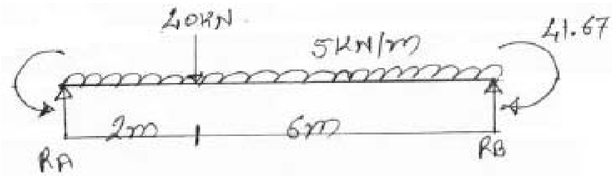
Net BM at C

$$= 90 - \left[41.67 + \left(\frac{71.67 - 41.67}{8}\times 6\right)\right]$$

$= 90 - 64.17$ [1 mark]

Net BM at C = 25.83 KN.m

(iii) Find support reactions to draw S.F.D



$\sum Fy = 0;\quad R_A + R_B = 80$ KN

$\sum M@A = 0;\quad -71.67 + (5 \times 8 \times 4) + (40 \times 2) + 41.67 - 8R_B = 0$

$\therefore\ R_B = 26.25$ KN

$\therefore\ R_A = 53.75$ KN [1 mark]

S.F. Calculations

S.F at left of A = 0

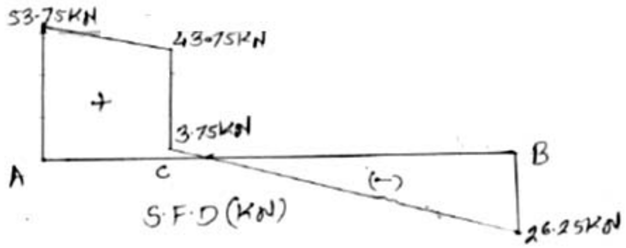
S.F at just right of A = $R_A$ = 53.75 KN

S.F at just left of C = 53.75 − 5 × 2 = 43.75 KN

S.F at just right of C = 43.75 − 40 = 3.75 KN [1 mark]

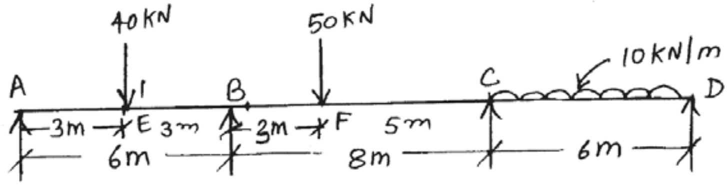
S.F at just left of B = 3.75 − 5 × 6 = − 26.25 KN

S.F at just right of B = − 26.25 + $R_B$ = 0 KN

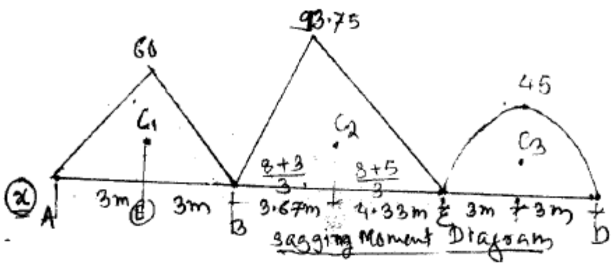



[1 mark]

**Q.6(c) A continuous beam is loaded as shown in Figure below. Find support moments and support reactions. Solve by three moment theorem only.** **[8]**




**Ans.:**




[1 mark]

Sagging moments

span AB = $\dfrac{40 \times (6)}{4}$ = 60 KN.m [½ mark]

Span BC = $\dfrac{50 \times 3 \times 5}{8}$ = 93.75 KN.m [½ mark]

Span CD = $\dfrac{10 \times 6^2}{8}$ = 45 KN.m [½ mark]

Area of moment diagrams

for span AB = $a_1 = \frac{1}{2}\times 6\times 60 = 180$ [½ mark]

for span BC = $a_2 = \frac{1}{2}\times 8\times 93.75 = 375$ [½ mark]

for span CD = $a_3 = \frac{2}{3}\times 6\times 45 = 180$ [½ mark]

Using Three moment theorem for
Pair.ABC

$$(M_A \times 6) + 2M_B(6 + 8) + (M_C \times 8) = \frac{-6(180\times 3)}{6}$$

As $M_A = 0$, (s/s at end) $- \dfrac{6(375)(4.33)}{8}$

$\therefore\ 28M_B + 8\,M_C = -\,1757.82$ … (1) [1 mark]

Pair B.C.D

$$(M_A \times 8) + 2M_B\,(8 + 6) + (M_D \times 6) = \frac{-6(375)(3.67)}{8}$$

$$= \frac{-6(180)(3)}{6}$$

As $M_D = 0$, (S/S at end)
$8M_B + 28\,M_C = -\,1572.19$ … (2) [1 mark]

Solving equation (1) and (2)
$M_B = -\,50.89$ kN.m (Hogging) [½ mark]
and $M_C = -\,41.61$ kN.m (Hogging) [½ mark]

**For support reaction**

(i) Take section at B and moment at left side of B
$(R_A \times 6) - (40 \times 3) = -\,50.89$
$\because\ R_A = +\,11.52$ kN

(ii) Take section at C and moment at Right side C

$$(R_D \times 6) - \left(10\times 6\times \frac{6}{2}\right) = -\,41.61$$

$\therefore\ R_D = +\,23.07$ kN [½ mark]

(iii) Take section at C and moment at Left side C
$(11.52 \times 14) - (40 \times 11) + (R_B \times 8) - (50 \times 5) = -\,41.61$
$R_B = +\,60.89$ kN

(iv) Using $\sum F_y = 0$ for overall beam
$R_A + R_B + R_C + R_D = 40 + 50 + (10 \times 6)$
$\therefore\ R_C = 54.52$ kN [½ mark]

❑ ❑ ❑ ❑ ❑