Vidyalankar

S.Y. Diploma : Sem. IV [CE/CS/CR/CV]

Theory of Structures

Prelim Question Paper Solution

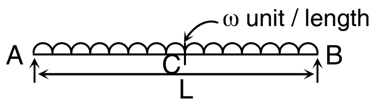
1. (a) (i) Direct Stress [1 mark]

It is defined as the ratio of direct load to the cross-sectional area and which gives the compressive stresses only.

$\therefore$ Direct stress = $\sigma_0 = \dfrac{\text{Direct load}}{\text{Cross} - \text{sectional area}}$ [1 mark]

$$\sigma_0 = \frac{\rho}{A}$$

1. (a) (ii)



Maximum slope = $\dfrac{dy}{dx} = \theta_A = \theta_B = \dfrac{wL^3}{24EI}$ [1 mark]

Maximum deflection = $y_{max} = y_c = \dfrac{5}{384} \cdot \dfrac{wL^4}{EI}$ [1 mark]

1. (a) (iii) The reaction between slopes. Deflection and radius of curvature. [2 marks]

$$\frac{1}{R} = \frac{(d^2y/dx^2)}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

1. (a) (iv)

Deflection at free end = $y_B = \dfrac{wL^3}{3EI}$ [2 marks]

1. (a) (v) Principle of Super Position [2 marks]

If the number of forces / moments are acting simultaneously an a body, then their combined effect an body is equal to the algebraic sum of the effect of the individual forces / moments considered separately.

1. (a) (vi) Carry Over Factor [2 marks]

The ratio of moment produced at a joint to the moment applied at the other joint, without displacing it is called as carry over factor.

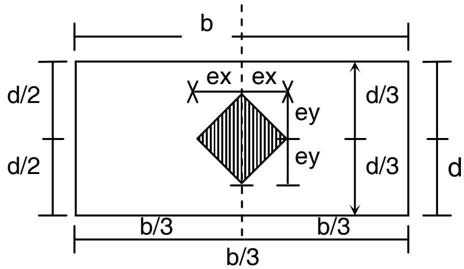
1. (a) (vii) Stiffness Factor [2 marks]

It is the moment required at end of beam to produce unit-rotation at that end without translation of either end

1. (a) (viii) Redundant frame ; $n > 2j - 3$ [1 mark]

Non-redundant frame ; $n < 2j - 3$ [1 mark]
where n = no of members j = no of joints

1. (b) (i) [2 marks]



[2 marks]

Incase of rectangular section, the external load it acts within middle third part of section, then no tension is produced any where in the section is called middle third rule.

OR

For no tension condition

$$e \le \frac{Z}{A}$$ [1 mark]

$$\le \frac{\frac{1}{6}bd^2}{bd}$$

$$e_y \le \frac{1}{6}d$$

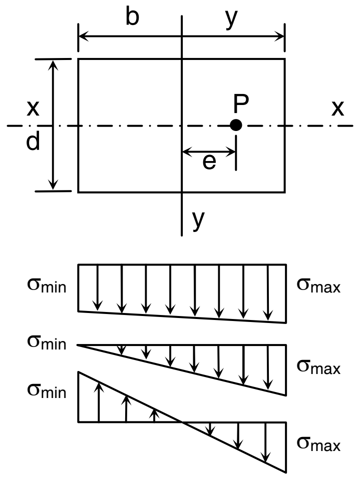
and $2e_y \le \frac{1}{3}d$

or $2e_x \le \frac{b}{3}$

[1 mark]

It means that the load can be eccentric, on either side of the geometrical axis, by an amount equal to d/6. Thus if the line of action of the load is within the middle third as shown by hatted area in figure, then there will be only compressive stress is known as middle third rate.

1. (b) (ii) [1 mark]

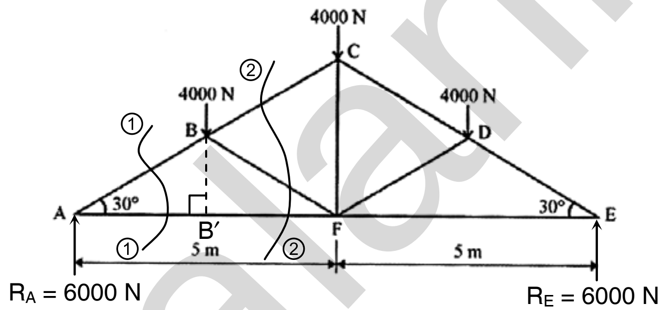


[1 mark]

[1 mark]

[1 mark]

1. (b) (iii)



(a) Support reactions
Due to symmetrical loading

$$R_A = R_E = \frac{\text{Total load}}{2} = \frac{4000 \times 3}{2} = 6000\,\text{N}$$ [1/2 mark]

(b) Geometrical properties
Length of member C.F.

CF = AF × tan 30 = 5 × tan 30 = 2.88 m

Length of member AC

$$AC = \sqrt{AF^2 + CF^2} = \sqrt{5^2 + 2.88^2} = 5.77\,\text{m}$$

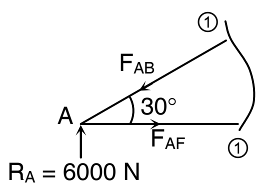
$$\therefore\ AB = BC = \frac{AC}{2} = \frac{5.77}{2} = 2.88\,\text{m}$$

Length of perpendicular BB′

BB′ = AB · sin 30 = 8.88 sin 30 [1/2 mark]

BB′ = 1.44 m

Length of AB′ = AB · cos 30
= 2.88 cos 30
AB′ = 2.50 m



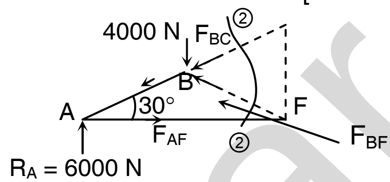
Consider section (1) – (1)

$\therefore \quad \Sigma t_y = 0$

$\therefore \quad F_{AB} \sin 30 = 6000$

$\therefore \quad F_{AB} = 12000$ N (Comp.)

[1 mark]



$\Sigma F_x = 0$

$F_{AB} \cos 30 = F_{AF}$

$F_{AF} = 10.392.304$ M (Tensile)

Consider (2) section (2) – (2)

Taking at moment @ F

$\therefore \quad \Sigma MF = 0$

$\therefore \quad 6000 \times 5 - 4000 \times 2.5 - F_{BC} \sin 30 \times 2.5$

$- F_{BC} \cos 30 \times 1.44 - F_{BF} \cos 30 \times 1.44 + F_{BF} \sin 30 \times 2.5 = 0$ [1 mark]

$20000 - 1.25\ F_{BC} - 1.25\ F_{BC} = 0$ [1 mark]

$$F_{BC} = \frac{20000}{2.50} = \underset{\text{(camp)}}{8000\text{ N}} = \underset{\text{(Due to symmetry)}}{F_{DC}}$$

OR

$\Sigma M_A = 00$

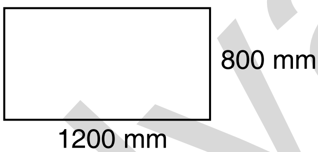
$4000 \times 2.5 - F_{BC} \cos 30 \times 1.44 + F_{BC} \sin 30 \times 2.5$

$- F_{BF} \cos 30 \times 1.44 - F_{BF} \sin 30 \times 2.5 = 0$

$10000 + 0 \times F_{BC} - 2.5\ F_{BF} = 0$

$\therefore \quad F_{BF} = \dfrac{10000}{2.5} = 4000$ N (Comp) [1 mark]

2. (a)



for rectangular section

$A = b \times d$

$$I = \frac{bd^3}{12}$$

$\therefore \quad e_x \le 1200/\sigma$

$e_x \le 200$ mm

By basic principle

$\sigma_d = \sigma_B$

$$\frac{P}{A} = \frac{My}{I}$$

$$\frac{P}{A} = \frac{P.e.y}{I}$$

$$\frac{1}{b \times d} = \frac{e \times d/2}{\left(\frac{b \times d^3}{12}\right)}$$

$$e = \frac{d}{\sigma}$$

Similarly $e \le b/\sigma$.

$e_y \le 800/\sigma$

$e_y \le 133.33$ mm

2. (b) $d_1 = 200$ mm $d_2 = 160$ mm $P = 6 \times 10^4$ N $e = 40$ mm

Total stress = Direct stress ± Bending stress

σ max, σ min = $\sigma_d + \sigma_B$

$$\sigma_d = \frac{P}{A} = \frac{6\times10^4}{\frac{\pi}{4}(200^2-160^2)} = 5.3 \text{ N/mm}^2$$

$$\sigma_B = \frac{My}{I} = \frac{P.e\,y}{I} = \frac{6\times10^4\times200/2\times40}{\pi/64\,[200^2-160^2]} = 5.175 \text{ N/mm}^2$$

$\sigma_{max} = \sigma_d + \sigma_B$

$= 5.3 + 5.175 = 10.47$ N/mm^2 [Compressive]

$\sigma_{min} = 5.3 - 5.175 = 0.125$ N/mm^2 [Compressive]

2. (c) h = 10 m

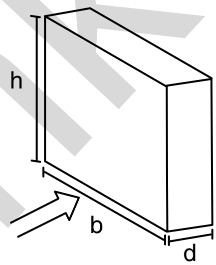
b = 3 m

d = 1.5 m

WP = 1.2 KN/m^2

Sp. Wt = 22 KN/m^2

Assuming wind $\perp$ to face hb



For max, min stress

σ max, σ min = $\sigma_d \pm \sigma_b$

$$\sigma_d = \frac{\text{sp. wt}\times\text{vol}}{A_{cs}} = \frac{22\times10^3\times10\times3\times1.5}{1.5\times3} = 22 \times 10^4 \text{ N/m}^2$$

$$\sigma_b = \frac{M.y}{I}$$

$$M = F \times \frac{n}{2} = 3.6\times10^4 \times \frac{10}{2}$$

$= 18 \times 10^4$ N/m^2

F → force due to wind pressure = A –A → Frontal Area as seen by how of wind k – coefficient of wind pressure

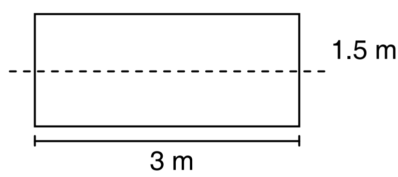
k = 1 → for rectangle = 2/3 → for circular.

F = WP × k × A

= 1.2 × 10^3 × 1 × 10 × 3

F = 3.6 × 10^4

$$\sigma_b = \frac{18\times10^4\times1.5/2}{\frac{3\times1.5^3}{12}}$$



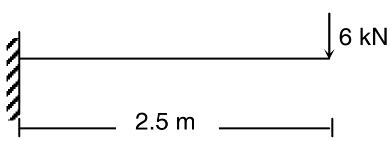
$\sigma_b = 16 \times 10^4$ N/m^2

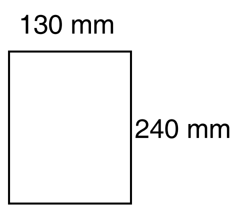
$\sigma_{max} = \sigma_d + \sigma_B$

$= 22 \times 10^4 + 16 \times 10^4 = 38 \times 10^4$ N/m^2

$\sigma_{min} = \sigma_d - \sigma_B = 22 \times 10^4 - 16 \times 10^4 = 6 \times 10^4$ N/m^2

2. (d)



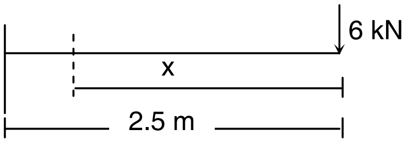


$E = 10^5\ N/mm^2 = 10^5 \times 10^3\ KN/m^2$

$I = \dfrac{130 \times 240^3}{12} = 149.76 \times 10^6\ mm^4$

$= 149.76 \times 10^{-6}\ m^4$

$\therefore\ EI = 14.976 \times 10^3\ KNm^2$



$\therefore\ M_x = -6x$

$EI\dfrac{d^2y}{dx^2} = -6x$

$EI\dfrac{dy}{dx} = \dfrac{-6x^2}{2} + C_1 = -3x^2 + C_1$

$EI\ y = \dfrac{-3x^3}{3} + C_1 x + C_2$

$EI\ y = -x^3 + C_1 x + C_2$

Allow at $x = 2.5$, $\dfrac{dy}{dx} = 0$

$\therefore\ O = \dfrac{-6}{2} \times 2.5^2 + C_1$ $\qquad \therefore\ C_1 = 18.75$

Now at $x = 2.5$, $y = 0$

$O = -2.5^3 + 18.75 \times 2.5 + C_2$ $\qquad \therefore\ C_2 = -13.25$

$\therefore\ EI\dfrac{dy}{dx} = -3x^2 + 18.75 \quad \rightarrow$ final slope eq.

$EI\ y = -x^3 + 18.75x - 31.25 \ \rightarrow$ final deflection eq.

For slope at free end., $x = 0$

$\therefore\ \dfrac{dy}{dx} = \dfrac{18.75}{EI} = 125 \times 10^{-3}$ rad.

For Deflection at free end., $x = 0$

$Y = \dfrac{-31.25}{EI} = 2.08 \times 10^{-3}$ m.

2. (e) Clapeyron's theorem of 3 moments for beam with same Moment of Inertia i.e. same cross section.

$$M_AL_1 + 2M_B(L_1 + L_2) + M_cL_2 = -\left[\frac{6a_1x_1}{L_1} + \frac{6a_2x_2}{L_2}\right]$$

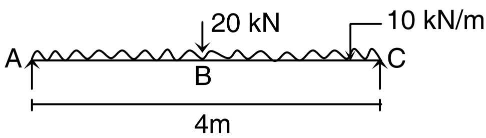
Clapeyron's theorem of 3 moments for Beam with different moment of Inertia i.e. diff. cross section.

$$M_A\frac{L_1}{I_1} + 2M_B\left[\frac{L_1}{I_1} + \frac{L_2}{I_2}\right] + M_C\frac{L_2}{I_2} = -\left(\frac{6a_1x_1}{L_1I_1} + \frac{6a_2x_2}{L_2I_2}\right)$$

$M_A \rightarrow$ moment at support A
$M_B \rightarrow$ moment at support B
$M_C \rightarrow$ moment at support C
$L_1 \rightarrow$ spam of part AB
$L_2 \rightarrow$ Spam of part BC
$I_1 \rightarrow$ MI of AB

$I_2 \rightarrow$ MI of BC
$a_1 \rightarrow$ Area of BMD for AB with hinged ends
$a_2 \rightarrow$ Area of BMD for BC with hinged ends
$x_1 \rightarrow$ C·C_1 of BMD for AB from Left end
$x_2 \rightarrow$ C·C_1 of BMD for BC from right end

2. (f)



$I_{xx} = 2 \times 10^8\ mm^4$
$= 2 \times 10^{-4}\ m^4$
$E = 2 \times 10^5\ N/mm^2$
$= 2 \times 10^8\ kN/m^2$
$\therefore\ EI = 4 \times 10^4\ kNm^2$

Due to symmetry of Beam and loads at midpoint.
Slope will be maximum at supports A and C.
Deflection will be maximum at mid point i.e. B.
Slope at A due to pt. load of 20 kN

$$\theta_A = \frac{WL^2}{16EI} = \frac{20 \times 4}{16\,EI} = \frac{20}{EI}$$

Slope at A due to U.D.L. of 10 kN/m

$$\theta_A = \frac{WL^3}{24\,EI} = \frac{10 \times 4^3}{24\,EI} = \frac{26.667}{EI}$$

$\therefore$ Total maximum slope at A = $\frac{20}{EI} + \frac{26.667}{EI}$

$$\theta_A = \frac{46.667}{EI} = 1.167 \times 10^{-3}\ rad$$

$\therefore$ Due to symmetry $\theta_B = -1.167 \times 10^{-3}$ rad

Deflection at pt B due to pt. load of 20 kN

$$y_B = \frac{WL^3}{48\,EI} = \frac{20 \times 4^3}{48\,EI} = \frac{26.667}{EI}$$

Deflection at pt. B due to UDL of 10 kN/m

$$y_B = \frac{5WL^4}{384EI} = \frac{5 \times 10 \times 4^4}{384\,EI} = \frac{33.33}{EI}$$

Total maximum deflection at pt. B

$$Y_{B\,max} = \frac{26.667}{EI} + \frac{33.33}{EI}$$

$$Y_{B\,max} = \frac{70}{4 \times 10^4} = 1.75 \times 10^{-3}\ m.$$

3. (a)
- Boundary condition for hinged end
  for hinged end y = 0 [1 mark]
  $\frac{dy}{dx} \neq 0$ [1 mark]

- Boundary condition for free end

$$y \neq 0$$ [1mark]

$$\frac{dy}{dx} \neq 0$$ [1 mark]

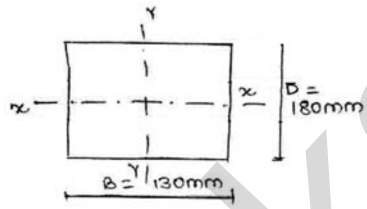
3. (b) L = 2m L$_1$ = 1m

$$I_{xx} = \frac{BD^3}{12}$$

$$= \frac{130 \times 180^3}{12}$$

$= 63.18 \times 10^6$ mm^4

$= 6.318 \times 10^{-5}$ m^4

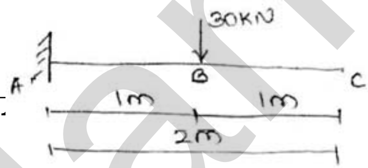


$$E = 105\ \frac{kN}{mm^2} = 105 \times 10^6\ \frac{kN}{m^2}$$

(a) Slope under point load. [1 mark]

$$Q_B = \frac{WL_1^2}{2EI}$$

$$= \frac{30 \times 1^2}{2 \times 10^5 \times 10^6 \times 6.318 \times}$$



$Q_B = 2.26 \times 10^{-3}$ ε ad [1 mark]

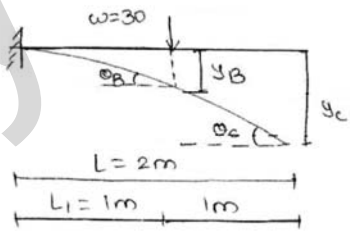
(b) Deflection under point load. [1 mark]

$$y_B = \frac{WL_1^3}{3EI} + \frac{WL_1^2}{2EI}(L - L_1)$$

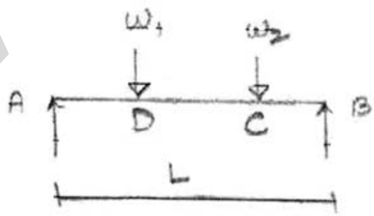
$$y_B = \frac{30 \times 1^3}{3 \times 6.6339 \times 10^3} + \frac{30 \times 1^2}{2 \times 6.6339 \times 10^3}(2 - 1)$$

$y_B = 3.768 \times 10^{-3}$ m

$y_B = 3.768$ mm [1 mark]

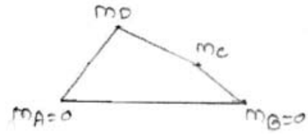


3. (c) ($W_1 > W_2$)



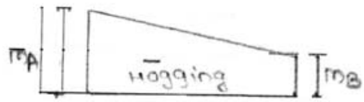
(a) Simply supported beam

[½ mark]



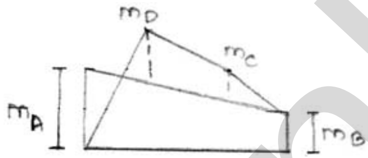
(b) Free B M diagram or μ diagram.

[½ mark]



(c) Fixed end moment diagram or 'μ' diagram.

[1 mark]



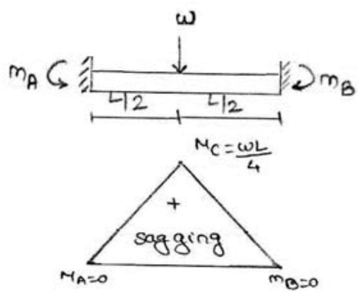
(d) Resultant Bm diagram (Combined effect of free and fixed end moment)

- Calculate the free moment by considering the beam to be simply supported. Hence draw free, BMD is positive (sagging) or is called as μ diagram BMD is plotted above the base line. [½ mark]
- Assume the simply supported beam is only subjected to fixed end moment. M_A and N_B which are negative (bogging) draw fixed end moment is μ diagram and very M_A at A and N_B at B. [½ mark]
- Super impose μ diagram over μ diagram and hence resultant BMD is drawn which is final BMD for fixed beam μ diagram is final BMD for fixed beam. μ diagram and μ diagram are opposite in nature there addition will be seen only. If they are plotted an same side of base line. [½ mark]
- Find net BMD at D and C by interpolation. [½ mark]

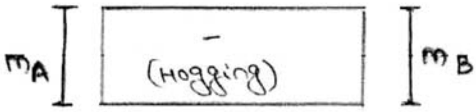
3. (d) Fixed end moment for fixed beam carrying point load at midspan.
Consider a fixed beam of span 'L' carrying a point load 'W' at midspan.

- Support reaction $V_A = V_B = \dfrac{W}{2}$
- Bending moment $M_A = 0$, $M_B = 0$, $M_C = \dfrac{WL}{4}$

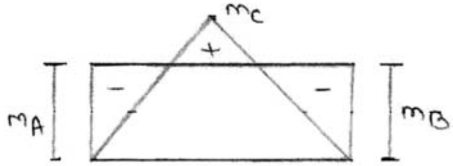
(a) [½ mark]



(b) μ diagram consider beam to be simply supported [½ mark]



(c) μ diagram for fixed end moment [1 mark]



(d) Net BMD
(Superimposition of μ and μ' diagram)
By first principle

$a = a'$ [1 mark]

$$\frac{1}{2}\times\frac{WL}{4}\times L = -M_A \times L$$

$$M_A = \frac{-WL}{8} = M_B$$ [1 mark]

3. (e) (a) Perfect truss: (Perfect frames)

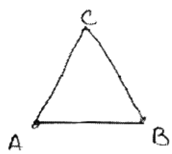
(1) A truss which does not collapse under the loading is called as perfect truss. [½ mark]

OR

When the number of members in frame is exactly equal to 2J-3 it is called as perfect frames.

(2) A truss in which the condition n = 2j-3 is satisfied is called as perfect frame
where n = number of member
J = number of joints [½ mark]

(3) Consider a triangle ΔABC in this case
n = 2J-3, n = 3, 2J-3 = 2(3) – 3 = 3
$\therefore$n is equal to (2J-3)
Hence it is called perfect truss. [1 mark]



(b) Imperfect truss:

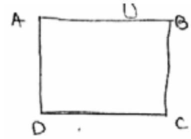
(1) A truss which collapse when loaded is called as unstable truss.

OR

When number of member in the frame are not equal to (2J-3) then it is called imperfect frame. [½ mark]

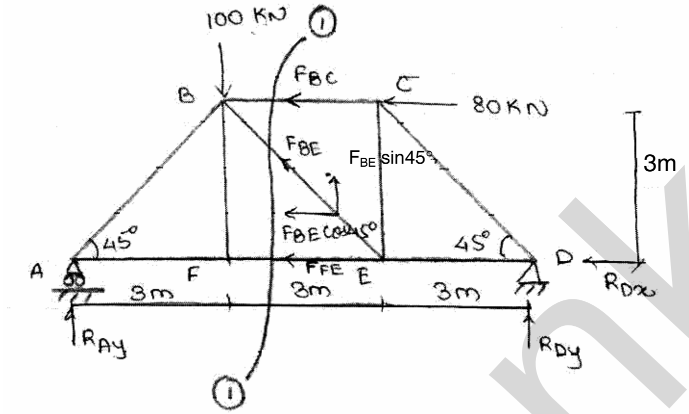
(2) In unstable truss condition n = 2J-3 is not satisfied. [½ mark]

(3) Consider a frame ABCD here
$n = 4$, $2J\text{-}3 = 2(4)-3 = 5$
$\therefore n \neq 2J\text{-}3$ hence it is called imperfect truss.



[1 mark]

3. (f)



$\Sigma Fx = 0$ $\quad -80 - R_{DX} = 0$
$R_{DX} = -80$ KN ($\leftarrow$)
$\Sigma Fy = 0$ $\quad R_{Ay} + R_{Dy} = 100$
$\Sigma N_A = 0$ $\quad +100 \times 3 - 80 \times 3 - R_{Dy} \times q = 0$
$300 - 240 = R_{Dy} \times q$
$R_{Dy} = 6.667$ KN
$R_{Ay} + R_{Dy} = 100$ [1 mark]
$R_{Ay} = 100 - 6.667$
$R_{Ay} = 93.333$ KN

To calculate forces in the member BC, BE, FE
Consider section (1) – (1) which cuts the member BC, BE, EF
Assume F_{FE}, F_{BC}, F_{BE} to be tensile in nature.
$\Sigma M_E = 0$ $\quad -F_{BC} \times 3 - 80 \times 3 - R_{Dy} \times 3 = 0$
$3F_{BC} = -240 -(6.667 \times 3)$
$3F_{BC} = -260.001$
$F_{BC} = -86.667$KN
$F_{BC} = 86.667$KN compression [1 mark]

To calculate F_{EE} take moment at B
$M_B = 0$ $\quad F_{FE} \times 3 - R_{Dy} \times 6 + R_{Dx} \times 3 = 0$
$3F_{FE} = 6R_{Dy} - 3R_{Dx}$
$3F_{FE} = (6.667 \times 6) - (3x - 80)$
$3F_{FE} = 280.002$
$F_{EE} = +93.334$KN
$F_{FE} = 93.334$KN Tensile [1 mark]

To calculate F_{BE} consider horizontal reactions

$\sum Fx = 0 - 80 - R_{Dx} - F_{BE}\cos45 - F_{BC} - F_{FE} = 0$

$$-80 - (-80) - (86.667) - 93.334 = F_{BE}\cos45$$

$$-6.667 = F_{BE}\cos45$$

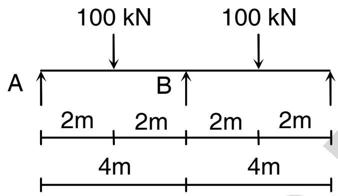
$$F_{BE} = -9.428KN$$

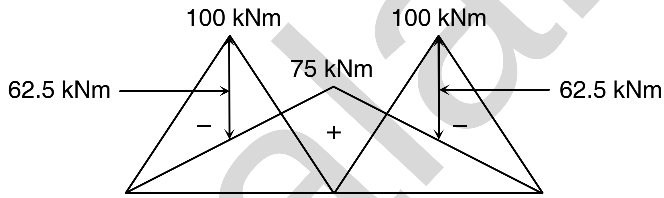
$$F_{BE} = +9.428KN \text{ compression}$$ [1 mark]

Force Table:

Sr. No.	Member	Magnitude	Nature
1	BC	86.667KN	Compression
2	FE	93.334KN	Tension
3	BE	9.428KN	Compression

4. (a)





[1 mark]

B.M.D. in kNm

$$M_AL_1 + 2M_B(L_1 + L_2) + M_CL_2 = \left(\frac{6a_1x_1}{L_1} + \frac{6a_2x_2}{L_2}\right)$$ [1 mark]

$$a_1 = \frac{1}{2}\times b\times h = \frac{1}{2}\times 4\times 100 = 200$$

$$x_1 = \frac{b_1}{2} = \frac{4}{2} = 2m$$

$$a_2 = \frac{1}{2}\times 4\times 100 = 200 \qquad x_2 = \frac{b_2}{2} = 2m$$ [1 mark]

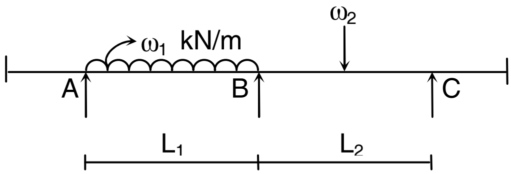
as M_A and M_C are simply supported hence $M_A = M_C = 0$

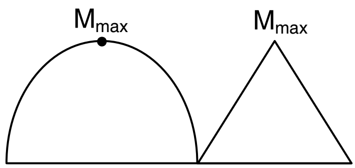
$$\therefore \quad 2M_B(4+4) = -\left[\left(\frac{6\times 200\times 2}{4}\right) + \left(\frac{6\times 200\times 2}{4}\right)\right]$$

$$16\,M_B = -\,1200$$ [1 mark]

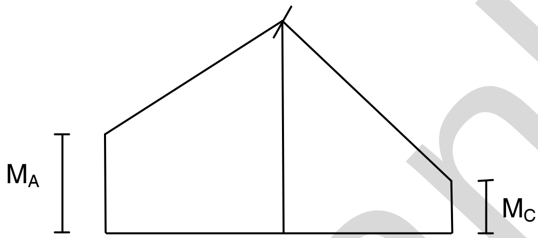
$$M_B = -\,75\text{ KnM}$$

4. (b)

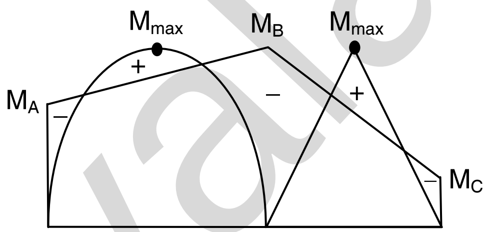




Free BM Diagram



BMD for support moments



[1 mark]

Net BMD

(i) Calpeyron's theorem of three moments with uniform EI for span AB and BC

$$M_AL_1 + 2M_B (L_1 + L_2) + M_CL_2 = -\left(\frac{6a_1\overline{x_1}}{L_1} + \frac{6a_2\overline{x_2}}{L_2}\right)$$ [1 mark]

where M_A = support moment at A
M_B = support moment at B
M_C = support moment at C
L_1 = Length of span at AB
L_2 = Length of span at BC
a_1 = area of free BMD for span AB
a_2 = area of free BMD for span BC
$\overline{x_1}$ = centroidal distance of free BMD over span AB from left end A
$\overline{x_2}$ = centroidal distance of free BMD over span BC from left end C

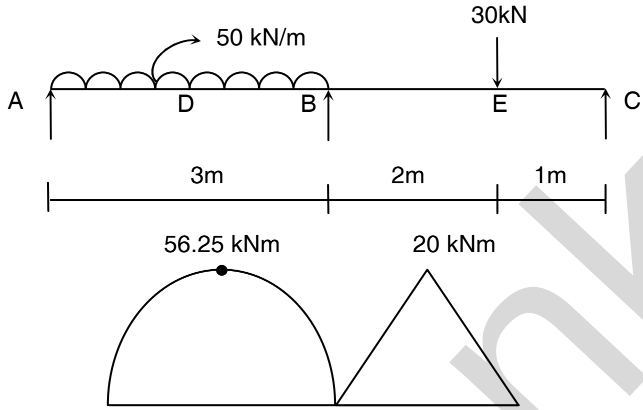
[1 mark]

(ii) Calpeyron's theorem of three moments with different MI

$$M_A \frac{L_1}{I_1} + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C \frac{L_2}{I_2} = -\left(\frac{6a_1\overline{x_1}}{L_1 I_1} + \frac{6a_2\overline{x_2}}{L_2 I_2}\right)$$ [1 mark]

where I_1 = moment of Inertia for span AB
I_2 = moment of Inertia for span BC

4. (c)



since A and C are simply supported

$M_A = M_C = 0$

For free BM span AB

$$M_{AB} = \frac{\omega_1^2}{8} = \frac{50\times 3^2}{8}$$ [1 mark]

$M_{AB} = 56.25$ kNm

For free BM span BC

$$M_{BC} = \frac{\omega_{ab}}{L} = \frac{30\times 2\times 1}{3}$$

For BM $M_{BC} = 20$ kNm

For span AB

$$a_1 = \frac{2}{3}\times b\times h = \frac{2}{3}\times 3\times 56.25 = 112.5 \text{ m}^2$$

$$x_1 = \frac{b}{h} = \frac{3}{2} = 1.5 \text{ m}$$ [1/2 mark]

For span BC

$$a_2 = \frac{1}{2}\times b\times h = \frac{1}{2}\times 3\times 20 = 30 \text{ m}^2$$

$$x_2 = \frac{L+b}{3} = \frac{3+1}{3} = 1.333\text{m}$$ [1/2 mark]

Apply Calpeyron's theorem of three moment for span AB and BC

$$M_A L_1 + 2M_B (L_1 + L_2) + M_C L_2 = -\left(\frac{\sigma a_1\overline{x_1}}{L_1} + \frac{\sigma a_2\overline{x_2}}{L_2}\right)$$

$$0 \times 3 + 2M_B\,(3 + 3) + 0 \times 3 = -\left(\frac{6 \times 112.5 \times 1.5}{3} + \frac{6 \times 30 \times 1.333}{3}\right) \qquad \text{[1 mark]}$$

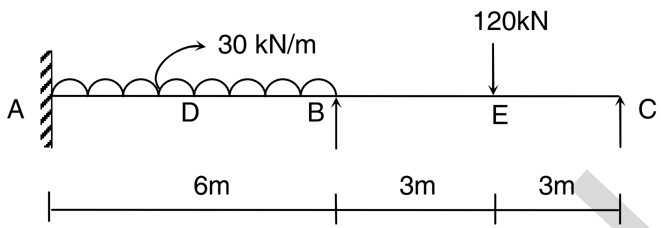
$$12M_B = -\,(337.5 + 79.98)$$

$$12M_B = -\,(417.48)$$

$$M_B = -\,34.79 \text{ kNm}$$

$M_A = 0$ $\quad$ $M_B = -\,34.79$ kNm $\quad$ $M_C = 0$ [1 mark]

4. (d)



For span AB

$$M_{AB} = \frac{-\omega l^2}{12} = \frac{-30 \times 6^2}{12} = -\,90 \text{ kNm}$$

$$M_{BA} = \frac{+\omega l^2}{12} = \frac{+30 \times 6^2}{12} = +\,90 \text{ kNm} \qquad \text{[1/2 mark]}$$

For span BC

$$M_{BC} = \frac{-\omega l}{8} = \frac{-120 \times 6}{8} = -\,90 \text{ kNm}$$

$$M_{BC} = \frac{+\omega l}{8} = \frac{+120 \times 6}{8} = +\,90 \text{ kNm} \qquad \text{[1/2 mark]}$$

Distribution factors [1 mark]

Joint	Member	Relative Stiffness	Total Stiffness	Distribution Factor
B	BA	$\frac{4EI}{\delta} = \frac{I}{6} = 0.1667\ I$	0.1667 I + 0.125 I	$\frac{0.1667\,I}{0.2917\,I} = 0.57$
	BC	$\frac{3EI}{\delta} = \frac{3I}{4 \times 6} = 0.125\ I$	= 0.2917 I	$\frac{0.125\,I}{0.2917\,I} = 0.43$

A	B		C
AB	BA	BC	CB
	0.57	0.43	
– 90	+ 90	– 90	+ 90
			– 90
		– 45	
– 90	+ 90	– 135	0
12.825 ←	+ 25.65	+ 19.35	
– 77.175	+ 115.65	– 115.65	0

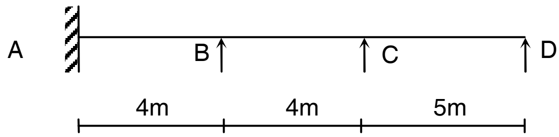
[1 mark]

[1 mark]

Final moments

$M_A = -77.175$ kNm
$M_B = 115.65$ kNm
$M_C = 0$

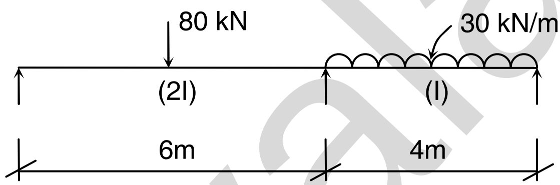
4. (e)



Distribution factor table

Joint	Member	Relative Stiffness	Total Stiffness	D.F.	
B	BA	$\frac{4EI}{4} = EI$	2 EI	0.5	[1 mark]
	BC	$\frac{4EI}{4} = EI$		0.5	[1 mark]
C	CB	$\frac{4EI}{4} = EI$	16 EI	0.625	[1 mark]
	CD	$\frac{3EI}{5} = 0.6EI$		0.375	[1 mark]

4. (f)



$M_{AB} = \frac{-\omega L}{8} = \frac{-80\times 6}{8} = -60$ kN, $\quad M_{BA} = \frac{+\omega L}{8} = \frac{+80\times 6}{8} = +60$ kN

$M_{BC} = \frac{-\omega L^2}{12} = \frac{-30\times 4^2}{12} = -40$ kNm, $\quad M_{CD} = \frac{+\omega L^2}{12} = \frac{+30\times 4^2}{12} = +40$ kNm

[1 mark]

Distribution factor table

Joint	Member	Relative Stiffness	Total Stiffness	D.F.	
B	BA	$\frac{3E\times LI}{6} = EI$	1.75 EI	0.57	[1/2 mark]
	BC	$\frac{3EI}{4} = 0.75$ EI		0.43	[1/2 mark]

Final Moment

A	B		C
AB	BA	BC	CB
	0.57	0.43	

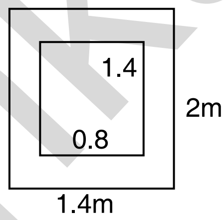
– 60	+ 60	– 40	+ 40	[1 mark]
+ 60			– 40	
	+ 30	– 20		
0	90	– 60	0	
	– 17.1	– 12.9		
0	+ 72.9	– 72.9	0	[1 mark]

$M_A = 0$
$M_B = 72.9$ kNm
$M_C = 0$

5. (a) Wind pressure $= 1.5$ kPa $= 1.5 \times 10^3$ pa

$\sigma_{max} = 280$ kN/m^2

Density of masonry $= 22$ kN/m^3



$$\sigma_{max} = \sigma_d + \sigma_D$$

$$\sigma_d = \frac{\text{Weight}}{\text{Area}} = \frac{\text{Density} \times \text{Volume}}{\text{Area}}$$

$$\sigma_d = \frac{22 \times 10^3 \times \text{Area} \times h}{\text{Area}}$$

$$\sigma_d = 22 \text{ x } 10^3 h$$

$$\sigma_d = \frac{My}{I}$$

$$\sigma_d = \frac{F \times h/2 \times y}{I}$$

F = Wind pressure $\times$ Area

$$= \frac{1.5 \times 10^3 \times 1.4 \times h \times h/2 \times 1}{0.75}$$

$= 1.5 \times 10^3 \times 1.4$ x h

$$\sigma_d = 1.4 \text{ x } 10^3 \, h^2$$

y = 2/2 = 1 m

$$280 \text{ x } 10^3 = 22 \text{ x } 10^3 h + 1.4 \times 10^3 \, h^2$$

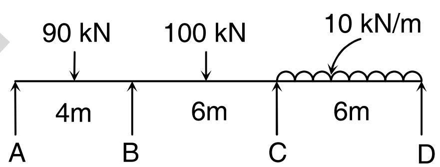
$$I = \frac{1.4 \times 2^3}{12} - \frac{0.8 \times 1.4^3}{12}$$

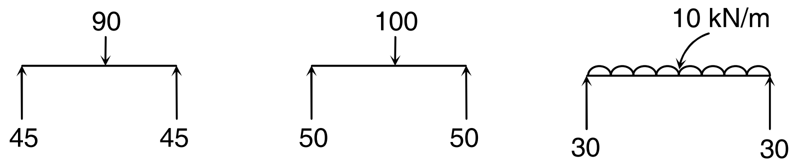
$= 0.75$ m^4

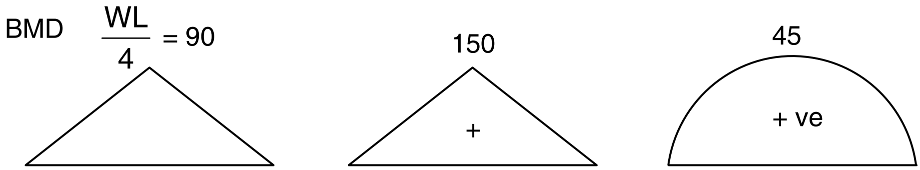
h = 8.32 m

$\sigma_{min} = 171.392$ kN/m^2 compressive

5. (b)







$a_1 = \frac{1}{2} \times 4 \times 90$ $\quad$ $a_2 = \frac{1}{2} \times 6 \times 150$ $\quad$ $a_3 = \frac{2}{3} \times 6 \times 45$

$a_1 = 180$ $\quad$ $a_2 = 450$ $\quad$ $= 180$

$x_1 = 2$ m $\quad$ $x_2 = 3$m $\quad$ $x_3 = 3$m

$L_1 = 4$m $\quad$ $L_2 = 6$m $\quad$ $L_3 = 6$m

By three moment theorem for AB and BC

$$\cancel{M_A}^{0} L_1 + 2\,M_B\,[L_1 + L_2] + M_C\,L_2 = -6\left[\frac{a_1x_1}{L_1} + \frac{a_2x_2}{L_2}\right]$$

$$2\,M_B\,[4 + 6] + M_C \times 6 = -6\left[\frac{180 \times 2}{4} + \frac{450 \times 3}{6}\right]$$

$$20M_B + 6M_C = -1890 \qquad \ldots (1)$$

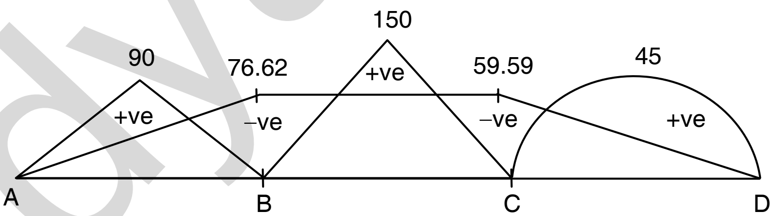
By three moment theorem for BC and CD

$$M_B\,L_2 + 2\,M_C\,[L_2 + L_3] + \cancel{M_B}^{0} L_3 = -6\left[\frac{a_2x_2}{L_2} + \frac{a_3x_3}{L_3}\right]$$

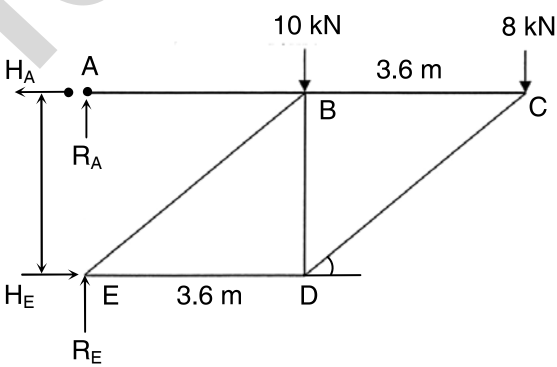
$$6\,M_B + 2M_C\,[6 + 6] = -6\left[\frac{450 \times 3}{6} + \frac{180 \times 3}{2}\right]$$

$$6\,M_B + 24\,M_C = -1890 \qquad \ldots (2)$$

$\therefore \quad M_B = -76.62$ kNm $\qquad M_C = -59.59$ kNm



5. (c)



$\sum F_y = 0$ $\qquad\qquad$ $\sum F_x = 0$

$\therefore \quad R_A + R_E = 18$ kN $\quad \ldots (1)$ $\qquad H_E - H_A = 0$

$\sum M_A = 0$

$3.2\, H_E = 10 \times 3.6 \times + 8 \times 7.2$

$H_E = 29.25$

$\therefore \quad H_A = 29.25$ kN

At Joint A

$R_A = 0 \qquad \therefore \quad R_E = 180$ kN

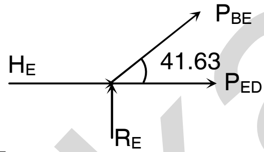
$\sum F_x = 0$

$\therefore \quad - H_A + P_{AB} = 0$

$P_{AB} = H_A$

$P_{AB} = 29.25$ kN

At Joint B



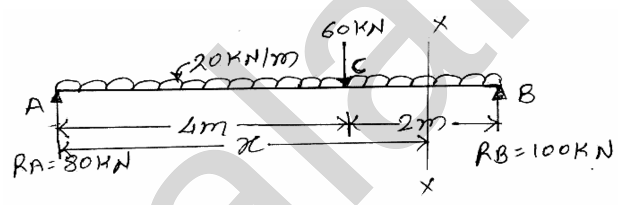
$\sum F_y = 0$

$R_E + P_{BE} \sin 41.63 = 0$

$$\therefore \quad P_{BE} = \frac{-RE}{\sin 41.63} = -27.09$$

$P_{BE} = -27.09$ kN

6. (a)



(i) Support reactions

$\sum Fy = 0; \quad R_A + R_B - 20 \times 6 - 60 = 0$

$R_A + R_B = 180$ KN

$\sum M@\ A = 0; (20 \times 6 \times 3) + (60 \times 4) - 6R_B = 0$

$\therefore \quad R_B = 100$KN

$\therefore \quad R_A = 180 - 100 = 80$ KN $\qquad$ [1 mark]

(ii) Consider a section x-x at a distance of x from A in portion CB

$$M_x = R_{Ax} - 20x.\frac{x}{2} \,\vdots\, 60\,(x-4)$$

$$\text{But EI } \frac{d^2y}{dx^2} = M_x = 80x - 10x^2 \,\vdots\, 60(x-4)$$ [1 mark]

$$EI\ \frac{d^2y}{dx^2} = 80x - 10x^2 \,\vdots\, 60(x-4) \qquad \ldots (1)$$

Integrated equation A w.r. to x, we get

$$EI = \frac{dy}{dx} = \frac{80x^2}{6} - \frac{10x^3}{3} \,\vdots\, 60\frac{(x-4)^2}{2} + C_1 \qquad \ldots (2)$$ [½ mark]

Again integrated equation B w.r. to x

$$EI\ y = \frac{80x^3}{6} - \frac{10x^4}{12} \,\vdots\, \frac{60(x-4)}{6} + C_1 x + C_2 \qquad \ldots (3)$$ [½ mark]

Apply boundary conditions

(a) At A, i.e. x = 0, y = 0 put in equation C

$EI\ (0) = 0 - 0 - 0 + 0 + C_2$

$C_2 = 0$ [½ mark]

(b) At B, i.e. x = 6, y = 0 put in equation C

$$EI\ (0) = \frac{80(6)^3}{6} - \frac{10(6)^4}{2} - \frac{60(6-4)^3}{6} + 6C_1$$

$$= 2880 - 1080 - 80 + 6C_1$$

$\therefore \quad C_1 = 286.67$ [½ mark]

Substitute the value of C_1 & C_2 in equation B & C respectively

$$EI\frac{dy}{dx} = \frac{80x^2}{2} - \frac{10x^3}{3} - \frac{60(x-4)^2}{2} - 286.67 \text{ ………… Slope equation}$$

$$EI\ y = \frac{80x^3}{6} - \frac{10x^4}{12} - \frac{60(x-4)^3}{6} - 286.67\,x \text{ …Deflection equation}$$

[1 mark]

(iii) Position of point of maximum deflection at maximum deflection slope remain zero i.e. at y_{max}, $\frac{dy}{dx} = 0$

Assume the maximum deflection will occur in portion AC

$$EI\ (0) = \frac{80x^2}{2} - \frac{10x^3}{3} - 286.67$$

$$0 = 40x^2 - 3.33x^3 - 286.67$$

Solving the above equation for x we get

x = 11.34m or x = −2.44m or x = 3.109m

$\therefore$ x = 3.109m

Assume portion for maximum deflection is comes as x < 4 [1 mark]

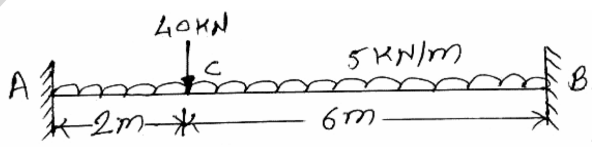
(iv) Maximum Deflection

$$EI\ y_{max} = \frac{80(3.109)^3}{6} - \frac{10(3.109)^4}{12} - 286.67 \times 3.109$$

$$EI\ y_{max} = 400.68 - 77.85 - 891.25$$

$\therefore \quad y_{max} = \frac{-568.42}{EI}$ [2 marks]

6. (b)



Let, a = 2m, b = 6m, L = 8m

(i) Support moments

$$M_A = -\left[\frac{wl^2}{12} + \frac{wab^2}{L^2}\right] = -\left[\frac{5 \times 8^2}{12} + \frac{40 \times 2 \times 6^2}{8^2}\right]$$

$M_A = -71.67$ KN.m [1 mark]

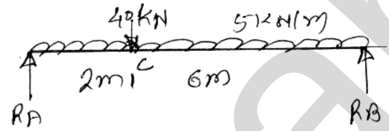
$$M_B = -\left[\frac{wl^2}{12}+\frac{Wa^2b}{L^2}\right] = -\left[\frac{5 \times 8^2}{12}+\frac{40 \times 2^2 \times 6}{8^2}\right]$$

$M_B = -\ 41.67$ KN.m

[1 mark]

(ii) Free B.M. ordinate below point load

a > reactions



$\sum F_y = 0$; $R_A + R_B = 80$ KN

$\sum M@A = 0$;

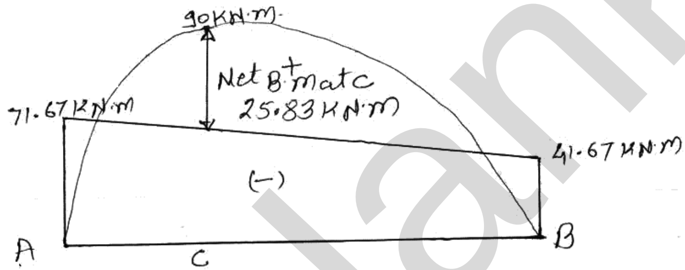
$(5 \times 8 \times 4) + (40 \times 2) - 8R_B = 0$

$\therefore$ $R_B = 30$KN

$\therefore$ $R_A = 50$KN

B mat C [1 mark]

$M_C = 50 \times 2 - 5 \times 2 \times 1 = 90$ KN.m



[1 mark]

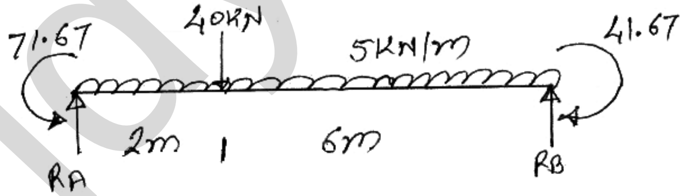
Net Bm at C

$$= 90 - \left[41.67 + \left(\frac{71.67 - 41.67}{8} \times 6\right)\right]$$

$= 90 - 64.17$ [1 mark]

Net Bm at C = 25.83 KN.m

(iii) Final support reactions to draw S.F.D.



$\sum Fy = 0$, $R_A + R_B = 80$ KN

$\sum M@A = 0$; $-71.67 + (5 \times 8 \times 4) + (40 \times 2) + 41.67 - 8R_B = 0$

$\therefore$ $R_B = 26.25$KN

$\therefore$ $R_A = 53.75$ KN [1 mark]

S.F. Calculations

S. F at left of A = 0

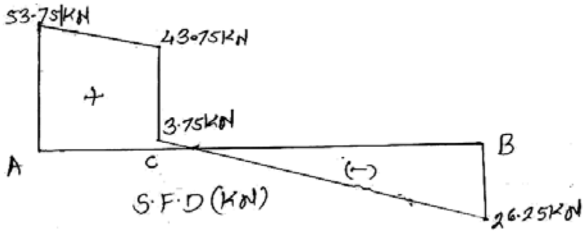
S. F at just right of A = R_A = 53.75 KN

S. F at just left of C = 53.75 – 5 x 2 = 43.75 KN

S. F at just right of C = 43.75 − 40 = 3.75 KN [1 mark]

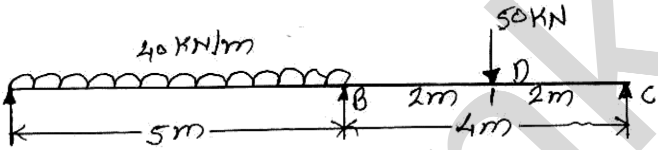
S. F at just left of B = 3.75 − 5 x 6 = −26.25 KN

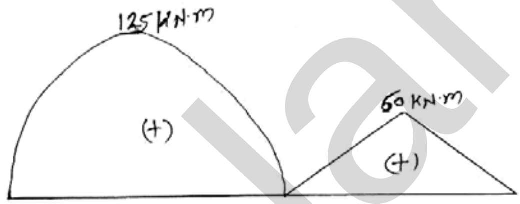
S. F at just right of B = −26.25 + R_B = 0 KN



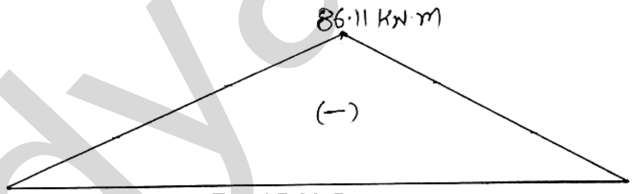
[1 mark]

6. (c)

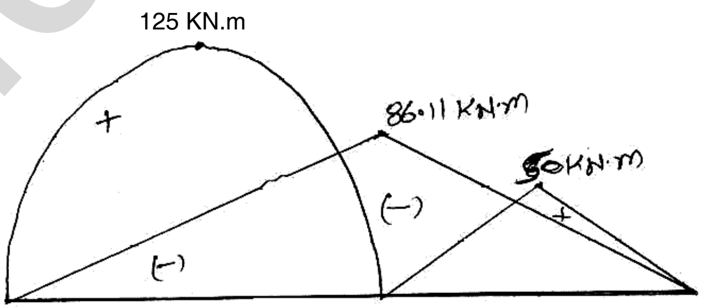




Free B.M. Diagram

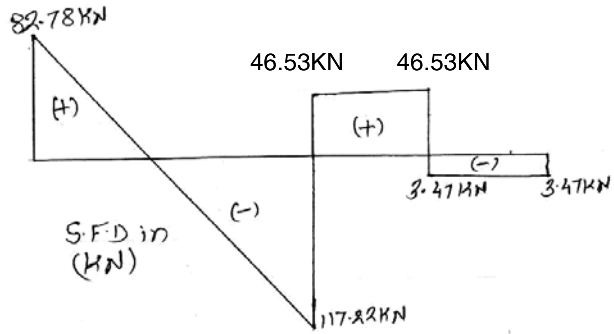


Fixed B.M. Diagram



[1 mark]

Final B.M. Diagram



[1 mark]

(i) Assume the beam is a series of s. s. beam& draw free BM diagram

span AB, max BM = $\frac{wl^2}{8} = \frac{40 \text{ x } 5^2}{8} = 125 \text{ KNm}$

span BC, max BM = $\frac{WL}{4} = \frac{50 \text{ x } 4}{4} = 50 \text{ KNm}$ [1 mark]

(ii) Apply clapeyron's theorem to span ABC

$M_A (L_1) + 2M_B (L_1 + L_2) + M_1 (L_1) = -\left[\frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2}\right]$ [1 mark]

Known moments

$M_A = M_C = 0$ …………… exterior simple support

$a_1 = \frac{2}{3} \text{x } 5 \text{ x } 125$ $a_2 = \frac{1}{2} \text{x } 4 \text{ x } 50$

$a_1 = 416.67$ $a_2 = 100$ [1 mark]

$\bar{x}_1 = \frac{5}{2} = 2.5$ $\bar{x}_2 = \frac{4}{2} = 2 \text{ m}$

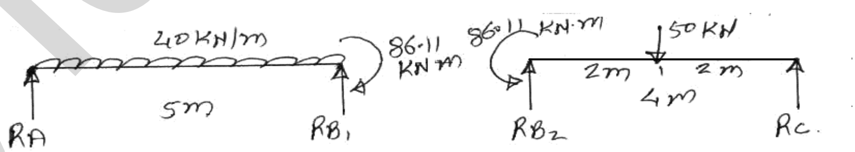
$L_1 = 5\text{m}$ $L_2 = 4\text{m}$

$2M_B (5 + 4) = -\left[\frac{6 \text{ x } 416.67 \text{ x } 2.5}{5} + \frac{6 \text{ x } 100 \text{ x } 2}{4}\right]$

$18M_B = -1550.01$

$\therefore$ $M_B = 86.11$ KN.m (Hogging) [1 mark]

(iii) Support reactions of C_1 of continuous beam



$\Sigma Fy = 0$, $R_A + R_B = 200$ KN $\Sigma Fy = 0;\ R_{B_2} + R_C = 50\text{KN}$

$\Sigma M@A = 0$ $\Sigma M@B = 0$

$(40 \text{ x } 5 \text{ x } 2.5) + 86.11 - 5R_{B_1} = 0$ $-86.11 + 50 \text{ x } 2 - 4R_C = 0$

$\therefore$ $R_{B_1} = 117.22$ KN

$\therefore$ $R_A = 82.78$ KN

$13.89 = 4R_C$

$R_C = 3.47$ KN

$\therefore$ $R_{B_2} = 46.53$ KN

$\therefore$ $R_A = 82.78 = 82.78$ KN

$R_B = R_{B_1} + R_{B_2} = 163.75$ KN [1 mark]

$R_C = 3.47 = 3.47$ KN

(iv) S.F. Calculations [1 mark]

S.F. at just left of A = 0

S.F. at just right of A = 82.78 KN (R_A)

S.F. at just left of B = 82.78 − 40 x 5 = −117.22 KN

S.F. at just right of B = −117.22 + R_B = 46.53 KN

S.F. at just left of D = 46.53 KN

S.F. at just right of D = 46.53 − 50 = −3.47 KN

S.F. at just left of C = −3.47

S.F. at just right of C = −3.47 + R_C = 0 KN

❑ ❑ ❑ ❑ ❑