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S.Y. Diploma : Sem. IV [AE/ME/MH/MI/PG/PT]

Theory of Machines

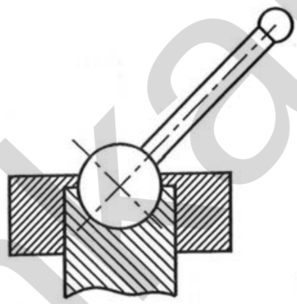
Prelim Question Paper Solution

1. (a) (i) Spherical Pair

When the two elements of a pair are connected in such a way that only one element (with spherical shape) turns or swivels about the other fixed element, the pair formed is called a spherical pair.

The ball and socket joint is a spherical pair.

Example : Attachment of a mirror for motorcycle, penstand.



1. (a) (ii) Kinematic Link or Element

It is defined as the part of a machine which has a relative motion with respect to some other part of same machine and the parts under consideration are connected to each other.
Example : Piston, Connecting rod, Crankshaft etc.

1. (a) (iii) 1) Absolute Velocity :

It is defined as the velocity of any point on a kinematic link with respect to fixed point.
Example, velocity of point A w.r.t. point O.
i.e. $V_{A/O} = V_A$

Similarly, velocity of point B w.r.t. point O $V_B/o = V_B$

2) Relative Velocity :

It is defined as the velocity of any point on a kinematic link w.r.t. some other point on the same link such that both the points are in motion.
Example, velocity of point B w.r.t. A.
i.e. $V_{B/A}$

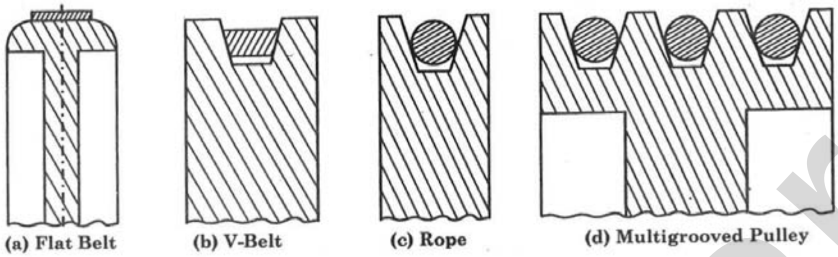
1. (a) (iv) 1) Pitch Curve :

While drawing the cam profile, it is assumed that the cam is at rest and the follower rotates around the cam at its relative velocity. The path traced by the trace point is called the pitch curve.

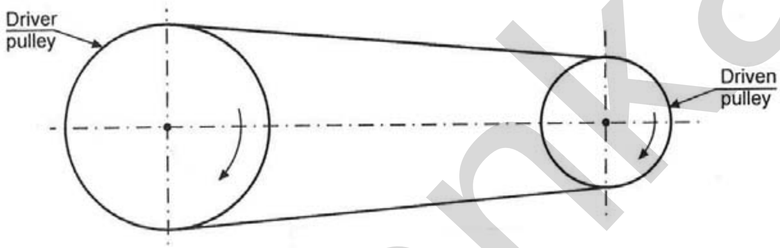
2) Prime circle :

It is the smallest circle drawn to the pitch curve from the centre of rotation of cam.

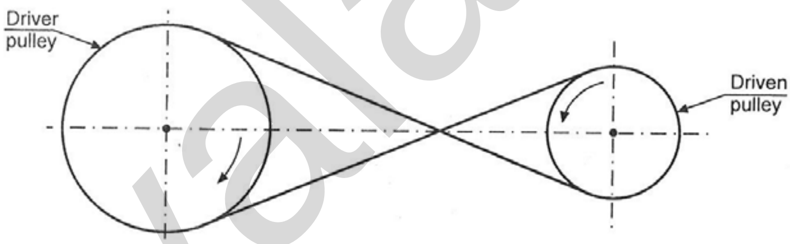
1. (a) (v) Types of Belts



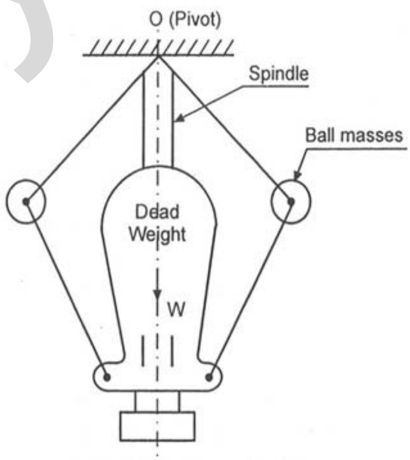
1. (a) (vi) Open Belt Drives



Cross Belt Drives



1. (a) (vii) Porter Governor



1. (a) (viii) Concept of Self-Locking and Self-Energizing of Brakes

In the shoe brake or band brake when the braking force required i.e. P is zero or negative i.e. no braking force is required to apply the brake, it is called as self-locking of the brake. When the force of friction helps to apply the brake, it is called as self-energizing brake.

1. (b) (i) Difference between Machine and Mechanism

	Machine	Mechanism
1)	The machine is a device to which energy is supplied in any form and the output is obtained in the form of useful mechanical work.	Mechanism is a kinematic chain in which one link is fixed.
2)	Machine can be a combination of number of mechanisms.	Mechanism is a combination of number of kinematic links.
3)	Machine is designed for transmitting motion as well as forces.	By and large mechanism is designed for transmitting motions.
4)	While designing the machine, physical parameters like stress, temperature etc. are considered.	While designing a mechanism generally lengthwise dimensions are considered.
5)	Examples : Lathe, milling, drilling, shaping, slotting, machines etc.	Examples : Typewriter, clock mechanisms etc.

1. (b) (ii) Inversions of a Single Slider Crank Chain

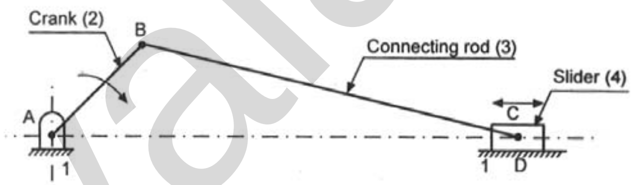


Fig. 1

Fig. 1 shows a basic single slider crank chain. It has three turning pairs and one sliding pair. The inversions of single slider crank chain are discussed as follows.

Reciprocating I.C. Engine Mechanism

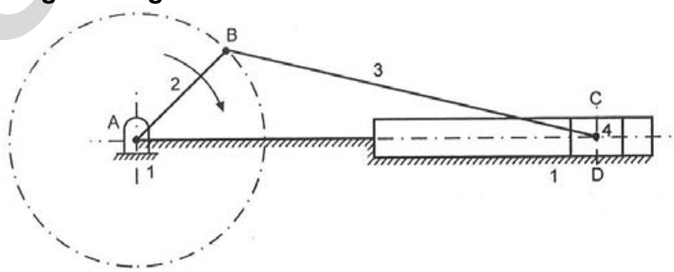


Fig. 2 : Reciprocating Engine Mechanism

This mechanism has four links and forming three turning pairs and one sliding pair. It converts rotary motion into reciprocating and vice-versa.

In a single slider crank chain as shown in Fig. 2, the links 1 and 2, links 2 and 3 and links 3 and 4 form three turning pairs while the links 4 and 1 form a sliding pair.

Link 1 - Cylinder and Frame of the engine (fixed link)
Link 2 - Crank
Link 3 - Connecting rod
Link 4 - Piston or Slider
As crank rotates, the slider reciprocates in the cylinder. When crank is a driver, then it is a reciprocating engine and when piston is a driver, then it is a reciprocating compressor.

1. (b) (iii) Scotch-Yoke Mechanism

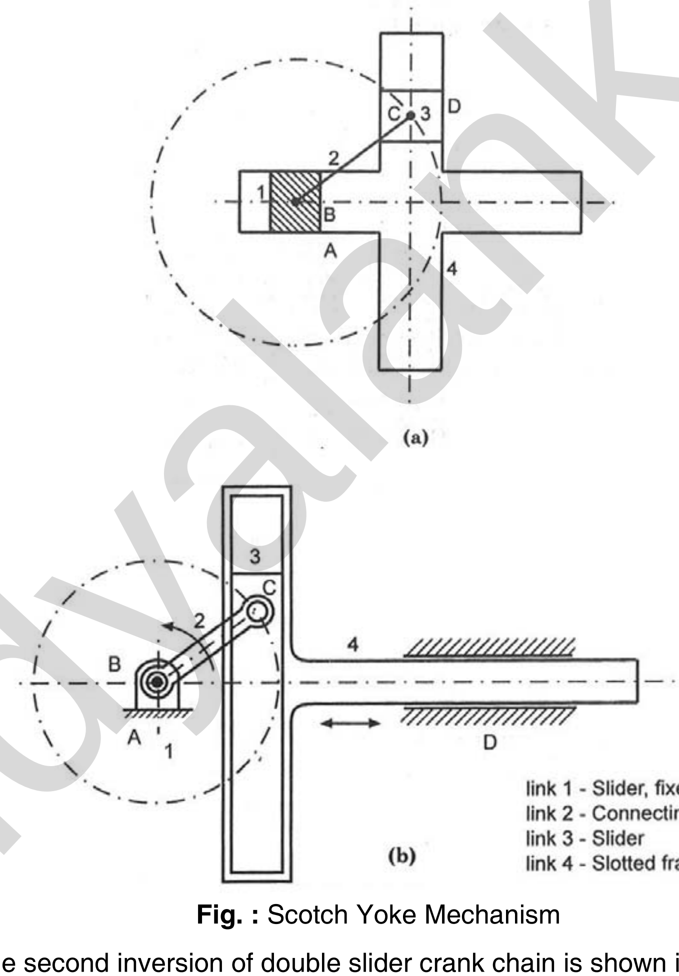


Fig. : Scotch Yoke Mechanism

- The second inversion of double slider crank chain is shown in Fig.
- In the inversion, one of the two sliders is fixed. Therefore, as the crank is rotated the slider will reciprocate and in turn, the whole frame will reciprocate.
- The frame will reciprocate perpendicular to the path of slider.

2. (a) Sprocket Mechanism of Bicycle

- Fig. shows the sprocket mechanism of a bicycle.
- It is a chain driven type mechanism.
- The driving sprocket is mounted (keyed) on the driving shaft A.
- It is rotated by the force applied on the pedal.
- The driven sprocket is keyed to the output shaft D.
- The rear wheel is mounted on the output shaft.
- As the driving sprocket rotates, the chain will also rotate, therefore, the driven sprocket will also rotate.
- Thus, the motion is transmitted to the rear wheel.
- It is a positive type of drive.
- It is used when the distance between two shafts is larger.

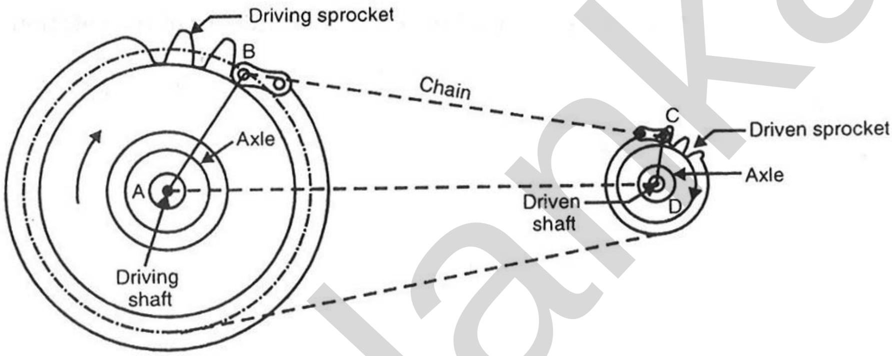
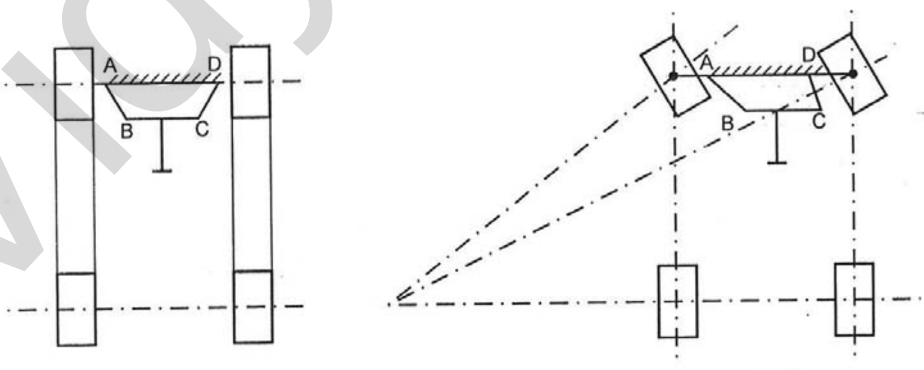


Fig. : Chain Driven Mechanism of a Bicycle

2. (b) Ackerman's Steering Gear Mechanism

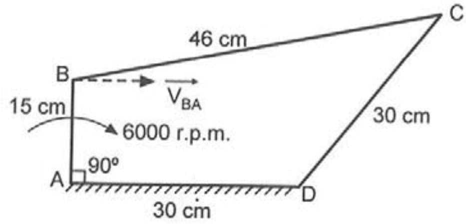
Figs. (a) and (b) show the working of Ackerman's Steering Gear. It consists of two cranks AB and DC connected by a link BC. AD is a fixed link.
As the vehicle takes the turn the inner wheel has to turn through larger angle. Outer wheel turns through smaller angle.
The cranks AB and CD rotate such that the axes of all the four wheels intersect in a point, as shown in Fig. (b).



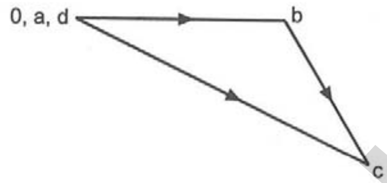
(a) **(b)**

Fig. : Ackerman's Steering Gear Mechanism

2. (c)



(a) Space diagram



(b) Velocity diagram

$$\omega_{BA} = \frac{2\pi N}{60} = \frac{2\pi \times 600}{60} = 20\ \pi \text{ radians/s.}$$

$$V_{BA} = \omega_{BA} \times \text{length } BA \quad = 20\ \pi \times 15 = 942.5 \text{ cm/s}$$

Since link AD is fixed, the velocity of points A and D is zero. Let it be represented by point 'O' in velocity diagram in Fig.

$\overrightarrow{V_{BA}}$ = 942.5 cm/s will be in the direction perpendicular to link BA and it will be represented by vector $\overrightarrow{ab}$. In velocity diagram, draw a vector $\overrightarrow{ab}$ = 942.5 cm/s on some chosen scale, say 1 cm = 250 cm/s, as shown in Fig. (b).

Now, point C is to be located on velocity diagram. Velocity $\overrightarrow{V_{CB}}$ = $\overrightarrow{bc}$ is only known in direction. It is perpendicular to link BC. Therefore, from point 'b', draw a line in the direction perpendicular to link BC in velocity diagram. Similarly, $\overrightarrow{V_{CD}}$ = $\overrightarrow{dc}$ is only known in direction, it is perpendicular to link CD. From point 'd', draw a line perpendicular to link CD in velocity diagram which cuts the direction of vector $\overrightarrow{bc}$ at 'c'. Hence, the velocity diagram is complete.

By measurement :

Absolute velocity of point C = $\overrightarrow{oc}$ or $\overrightarrow{ac}$ or $\overrightarrow{dc}$
= length dc × scale
= 5.1 × 250 = 1275 cm/s

$$\omega_{CB} = \frac{\overrightarrow{V_{CB}}}{\text{length } CB} = \frac{\overrightarrow{bc}}{\text{length } CB}$$

$$= \frac{\text{length } bc \times \text{scale}}{\text{length } CB}$$

$$= \frac{2.55 \times 250}{46} = 13.859 \text{ rad./s}$$

$$\omega_{CD} = \frac{\overrightarrow{V_{CD}}}{\text{length } CD} = \frac{\overrightarrow{dc}}{\text{length } CD}$$

$$= \frac{1275}{30} = 42.5 \text{ rad./s}$$

2. (d)

Followers

- According to Follower Shape
- According to Motion
- According to Line of Stroke of Follower

(i) According to Shape :

The followers are classified into four types according to shape of follower as :

1. Knife Edged follower.
2. Flat Face or Mushroom Follower.
3. Roller Follower
4. Spherical Follower

These followers are shown in Fig. 1

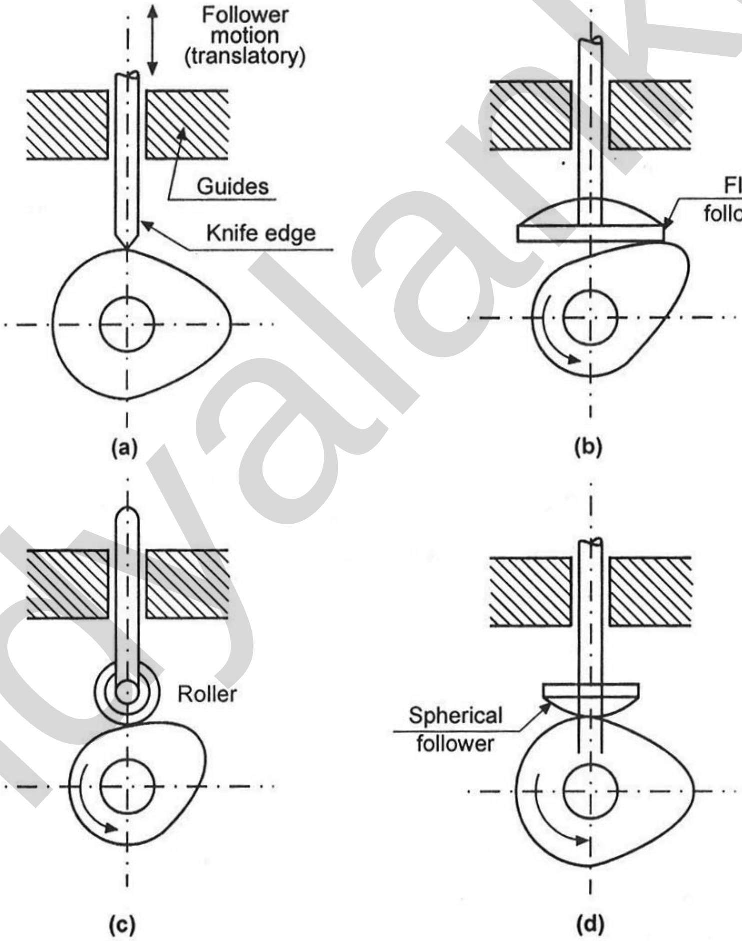


Fig.: 1

1. Knife edged followers are simple in construction and the motion between the cam and follower is of sliding type.

2. In roller followers, the motion between the cam and follower is of rolling type. It reduces the wear to a considerable extent.
3. The flat followers are preferred over the roller followers when the space is limited.

(ii) According to Type of Motion of Follower :

There are two types of followers according to this criteria.

1. Reciprocating Follower   2. Oscillating Follower

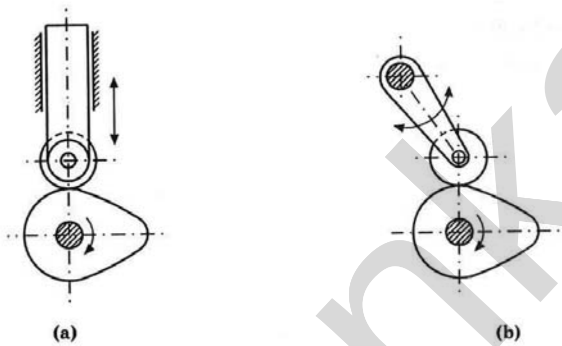


Fig. : 2

1. ***Reciprocating Follower*** : Fig. 2(a) shows a reciprocating follower, in which the cam rotates and the follower reciprocates or translates in the guides.
2. ***Oscillating Follower*** : Fig. 2(b) shows a oscillating follower, in which the cam rotates and the follower oscillates about a suitable pivot on the frame.

(iii) According to Line of Motion of Follower :

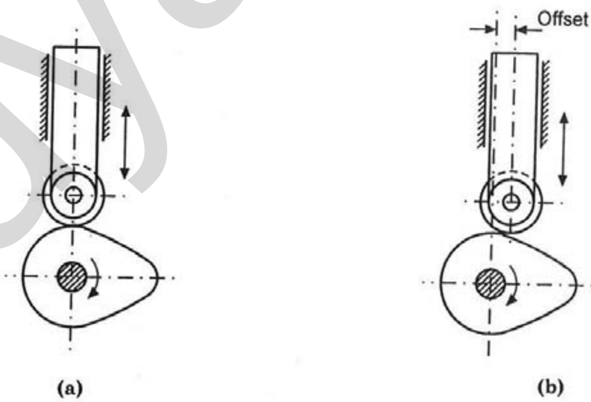
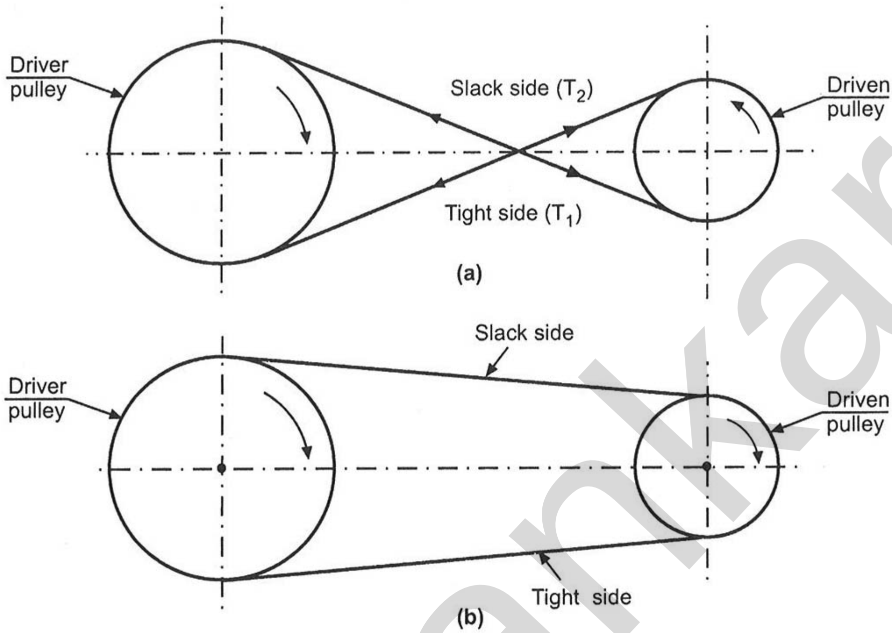


Fig.3

1. ***Radial Follower*** **:** The line of motion of follower passes through the axis of the cam Figure 3(a).
2. ***Offset Follower*** **:** In this follower, the line of motion of follower is away (offset) from the axis of the cam Figure 3(b).

2. (e) Creep in Belt



When the power is transmitted by the belt apart from the initial tightening of the belt the driving tensions are generated in the side of belt due to friction between the belt and the pulley surface. The side of belt leaving the driven pulley and approaching the driver pulley is pulled by the driver pulley. Hence, it has more tension than the other side. Therefore, the side of belt approaching the driver pulley is called as the tight side (T_1).

The side of the belt leaving the driver pulley has less tension therefore, it is called as slack side (T_2).

The length of belt approaching the driver pulley is more than length of belt leaving the driver pulley. It causes a relative motion of the belt on the pulley. This relative motion is called as creep in belts.

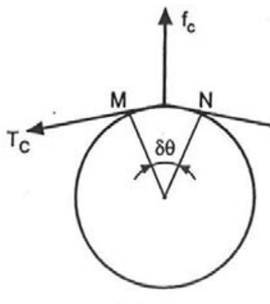
Considering the creep the velocity ratio is given by,

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \left(\frac{E + \sqrt{f_2}}{E + \sqrt{f_1}} \right)$$

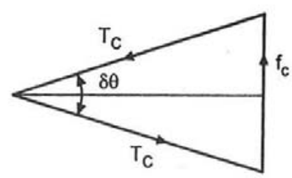
where, E = Young's modulus for the material of the belt
f_1 = Stress in the belt material on the tight side
f_2 = Stress in the belt material on the slack side

2. (f) Centrifugal Tension

As the belt runs at a uniform speed over an arc of a circle the centrifugal force acts on the belt due to its mass. To balance this centrifugal force, a tension is generated equally in tight and slack side of the belt. It is called as centrifugal tension (T_c).



(a)



(b)

Let us consider a small element of belt subtending angle dθ as shown in Fig. (a)

Let m = Mass of belt per metre length in kg
r = Radius of pulley in m
V = Linear velocity of belt in m/s
f_c = Centrifugal force in newtons
T_c = Centrifugal tension in newtons

Therefore, the length of element of belt MN = $r\delta\theta$

∴ Mass of element of belt MN = $mr\delta\theta$

∴ The centrifugal force acting on the element of belt MN

$$f_c = mr\delta\theta\frac{V^2}{r} = mV^2\,\delta\theta$$

Now, referring the force polygon in Fig. (b) and resolving the forces vertically.

$$T_c \sin\frac{\delta\theta}{2} + T_c \sin\frac{\delta\theta}{2} = f_c$$

$$2T_c \sin\frac{\delta\theta}{2} = mV^2\,\delta\theta$$

∵ $\delta\theta$ is very small,

$$\sin\frac{\delta\theta}{2} \approx \frac{\delta\theta}{2}$$

∴ $2T_c\,\frac{\delta\theta}{2} = mV^2\delta\theta$

∴ $T_c = mV^2$

3. (a) Comparison between Flat Belt Drive and V-Belt Drive

	Flat Belt Drive	V-Belt
1)	Suitable for moderate power transmission when the distance between the shafts is large.	Suitable for high power transmission when the distance between the shafts is small.
2)	Requires large space due to large centre to centre distance between shafts.	Gives compactness due to small distance between the shafts.

3)	Due to less frictional grip, the belt may slip from the pulley. Hence, it is not a positive drive.	Due to groove, more frictional grip between belt pulley. Hence tendency of slipping is less. So it is a positive drive.
4)	High velocity ratio may not be obtained.	High velocity ratio may be obtained.
5)	Power transmission by flat belt is less than that by V belt for the square value of co-efficient of friction, angle of lap and allowable tension.	Due to wedging action of the belt in the groove, high value of limiting ratio of tension is obtained. As $\frac{T_1}{T_2} = e^{\mu\theta \text{ cosec } \alpha}$ Hence, power transmission is more compared to flat belts.
6)	Flat belts cannot be operated in both directions.	V belts may be operated in either direction with tight side at the top or bottom. The centre line may be vertical, horizontal or inclined.

3. (b) Given : b = 20 cm; t = 0.8 cm; V = 1000 m/min = $\frac{1000}{60} = \frac{50}{3}$ m/s;

$\rho = 0.001$ kg/cm^3; Stress, f = 200 N/cm^2

and $\frac{T_1}{T_2} = 2$

Mass of the belt per metre length of the belt,

$m = \rho.b.t.\ell = 0.001 \times 20 \times 0.8 \times 100$

$= 1.6$ kg/m length

Centrifugal tension, $T_C = mV^2$

$$= 1.6 \times \left(\frac{50}{3}\right)^2 = 444.4 \text{ N}$$

Maximum permissible pull in the belt,

$T_{max} = f.b.t = 200 \times 20 \times 0.8 = 3200$ N

$T_1 = T_{max} - T_C$

i.e. $= 3200 - 444.4 = 2755.6$ N

$$\text{Maximum power, } P = \frac{T_1\left(1-\frac{T_2}{T_1}\right)v}{1000} = \frac{2755.6\left(1-\frac{1}{2}\right)\frac{50}{3}}{1000} = 22.96 \text{ kW}$$

3. (c) Given : d = 60 mm, $\mu = 0.25$, N = 200 r.p.m., $\theta = 180 \times \frac{\pi}{180} = \pi$

$\therefore$ $T_{max} = 2500$ N

For maximum power transmission

$$T_1 = \frac{2}{3}T_{max} = \frac{2}{3} \times 2500$$

$= 1666.67$ N

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\therefore \quad T_2 = \frac{1666.67}{e^{(0.25\times\pi)}} = 759.89 \text{ N}$$

$$\therefore \quad V = \frac{\pi dN}{60} = \frac{\pi\times 60\times 10^{-3}\times 200}{60}$$

$$\therefore \quad V = 0.628 \text{ m/s}$$

Power transmitted

$$P = (T_1 - T_2)\,V$$
$$= (1666.67 - 759.89) \times 0.628$$
$$= 569.75 \text{ W}$$

3. (d) Turning Moment Diagram for an Internal Combustion Engine

The indicator diagram for single cylinder four stroke petrol engine is shown in Fig.1

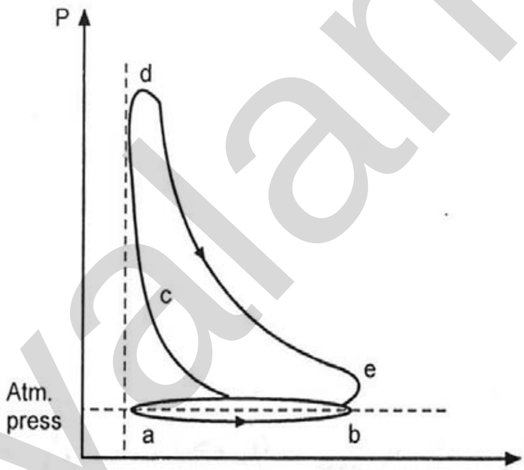


Fig. 1

The various strokes of the engine are :

(i) Suction : The piston draws the.mixture of fuel and air during its outward stroke. The pressure inside the cylinder is less than the atmosphere as represented by the curve ab.

(ii) Compression : During the next stroke the piston compresses the mixture upto the clearance volume of the cylinder as shown by the curve bc.

(iii) Expansion : At the end of compression the mixture is ignited and the combustion takes place nearly at constant volume. The pressure and temperature of the gas rises upto 'd' and then it expands in its outward stroke upto 'e'. The work is done by the gases on the piston.

(iv) Exhaust : Burnt gases are expelled to the atmosphere during the inward stroke of the piston.

The turning moment on the crank shaft for different crank angles can be calculated in the same way as in the case of steam engine. The only difference is that in a four stroke IC engine the diagram repeats after two revolutions instead of one revolution.

The torque diagram for gas forces is shown by dotted lines in Fig. 2. The pressure on the gases during suction stroke is slightly less than atmospheric due to which there is very small negative loop. During compression stroke, the negative loop is obtained because the work is done by the piston on the gases. During expansion stroke, we get a positive large loop since the work is done by the gases on the piston while the work is done on the gases by the piston during exhaust stroke due to which we again set a small negative loop. It should be noted that the pressure of the gases inside the cylinder is slightly higher than atmospheric.

The torque diagram for inertia forces have been shown by long broken lines. After superimposing the inertia torque on gas torque, the net turning moment diagram is obtained as shown by firm lines in Fig. 2

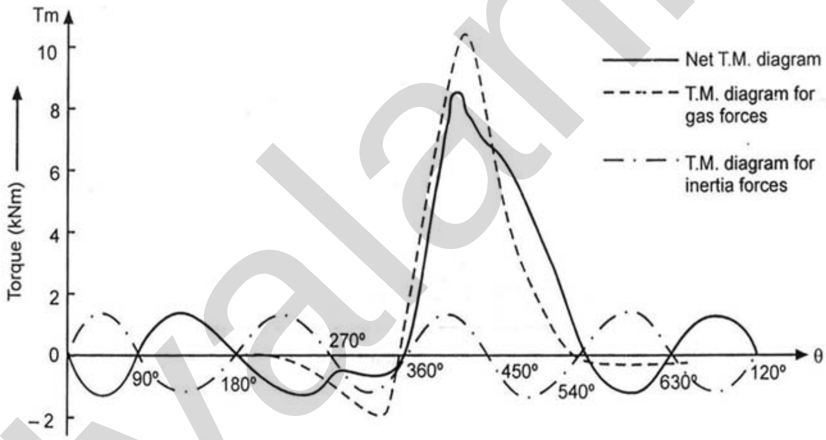


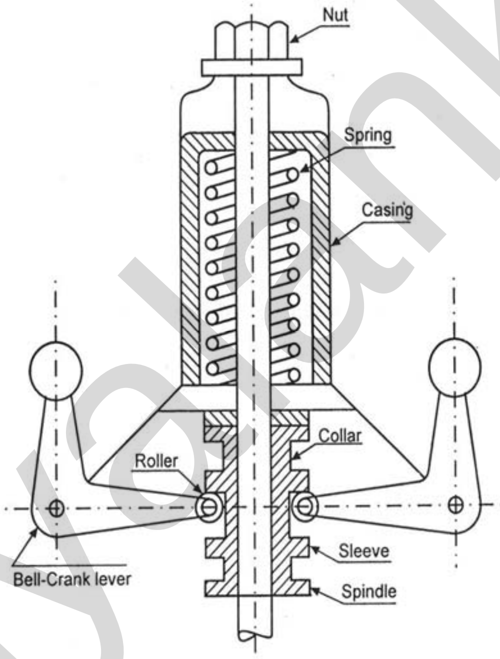
Fig.: 2

3. (e) Differentiate between flywheel and governor

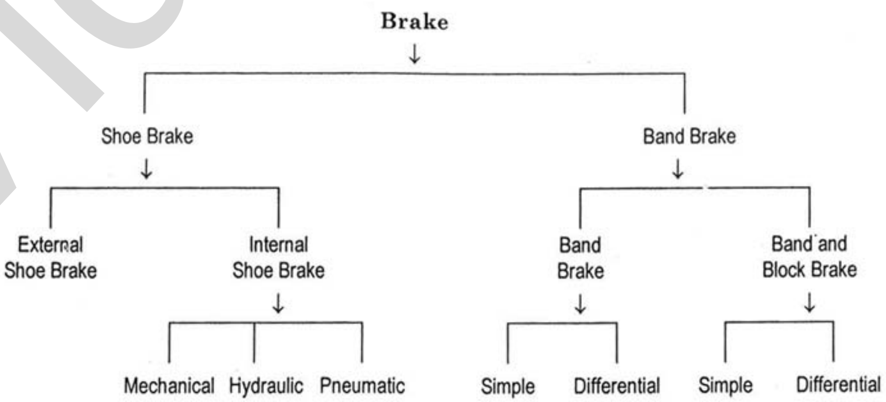
	Flywheel	Governor
1)	The function of flywheel is used to control the speed vibrations of turning moment during cycle.	It is used to keep the variation in mean speed of engine within prescribed limit due to fluctuations in load over period of time.
2)	Flywheel controls $\frac{\delta N}{\delta t}$.	Governor controls δN.
3)	A flywheel stones the energy and gives up the energy whenever required during cycle.	It regulates the speed by regulating the quantity of charge of the prime mover.
4)	It regulates speed during one cycle only.	It regulates the speed over a period of time.
5)	A Flywheel has no control over quantity of change.	A governor takes case of quantity of working fluid.

3. (f) Hartnell Governor

- Fig. shows a spring loaded Hartnell governor.
- It has a speed casing keyed to the spindle.
- The spindle is connected to the engine shaft through the bevel gear.
- A sleeve is mounted on the spindle which rotates alongwith the spindle as well as slides along the spindle.
- A compressed spring is mounted in the casing between the top of the casing and top of the sleeve.
- The ball masses are mounted on the bell crank lever.
- In this governor, the controlling force is provided due to spring force and weight of the sleeve.
- The ball masses can fly away or come closer with the help of bell crank lever.
- Working of the Hartnell governor is same as the working of simple governor.



4. (a) Classification of Brakes



4. (b) Given : $r_b = \frac{40}{2}$ = 20 cm Or 0.2 m; T_b = 1500 N.m; a = 10 cm; b = 2 cm; μ = 0.3;

$\theta = 225 \times \frac{\pi}{180}$ radians; $\ell = 60$ cm

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{T_1}{T_2} = e^{0.3\left(\frac{\pi}{180}\times 225\right)} = 3.248 \qquad \ldots(i)$$

But, $T_b = (T_1 - T_2)\, r_b$

Or $1500 = (T_1 - T_2)\, 0.2$

Or $T_1 - T_2 = 7500$ N …(ii)

$\therefore$ $3.248\, T_2 - T_2 = 7500$

Or $T_2 = 3336.3$ N and $T_1 = 10836.3$ N

Taking moments about the fulcrum :

$T_1.a = T_2.b + P.\ell$

$10836.3 \times 10 = 3336.3 \times 2 + P \times 60$

Or $P = 1694.8$ N

4. (c) INTERNAL EXPANSION SHOE BRAKES

Mechanical Breaking System

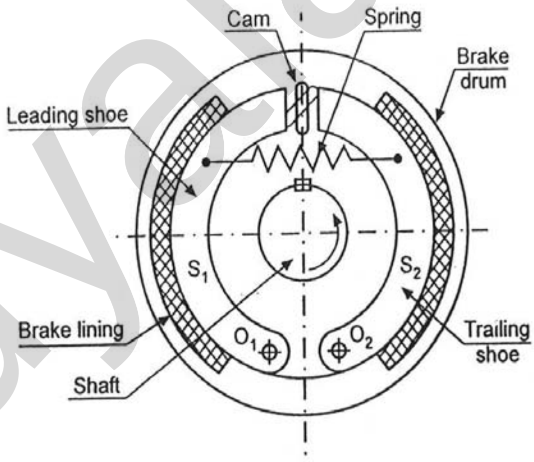


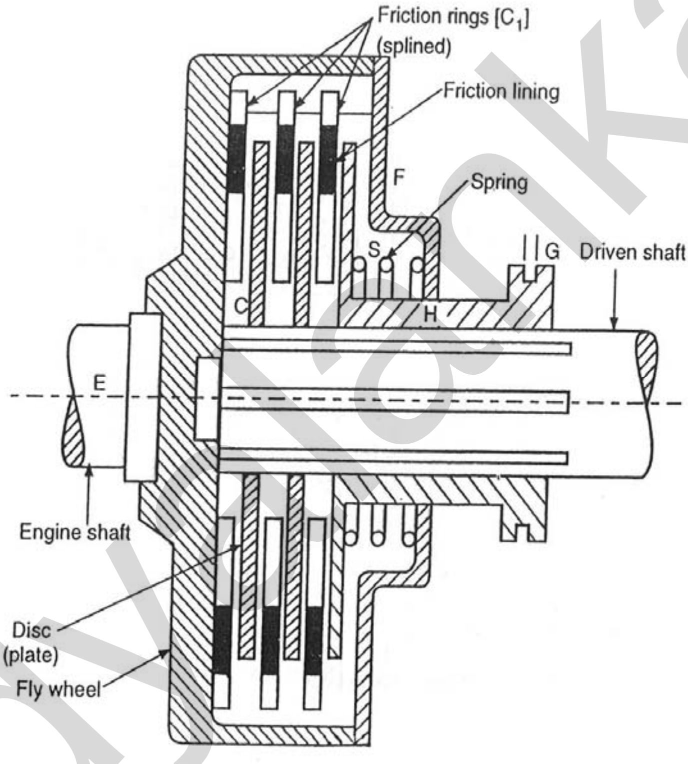
Fig. shows the mechanical brakes used in automobiles. This braking system is generally used for parking brakes. It consists of two brake shoes which are mounted inside the brake drum. The shoes are pivoted at their heels.

A cam is mounted between the two brake shoes. It is connected to the brake pedal through mechanical linkages. The brake liners made of ferrodo are mounted on the brake shoes. A retrieving spring is mounted between the brake shoes.

As the brake pedal is pressed, the mechanical linkages rotate the cam so that it applies the force on the toes of the brake shoes. The brake shoes go away from each other and the liners are pressed against the brake drum and the brake is applied. As the brake pedal is released the cam rotates back to initial position and the retrieving spring brings the brake shoes back to the initial position and braking force is released.

4. (d) Multiple Plate Clutch

This type of clutch is used for transmitting power in automobiles like scooters. These are also used in industries like textile and pater industries. The diagram of a multiple plate clutch has been shown in Fig.



A is the driving shaft which is fixed to the flywheel and the outer casing J. This casing has internal grooves on which the friction plate - C (few in numbers) are fixed and alternately the friction rings C_1 are fixed to the boss H. F is the pressure plate, 'S' the spring and G represents the operating lever which is integral to pressure plate. E represents the driven shaft.

The working of this clutch is similar to the working of single plate clutch explained earlier.

When the applied force from the operating lever is removed, the springs will exert an axial force on the pressure plate F which slides in the axial direction on the shaft E. Due to this, the friction plates C-C come in contact with friction plates $C_1 – C_2$ through friction material. This transmits the required frictional torque and the driven shaft rotated at the speed of driving shaft.

4. (e) Given : N = 150 rpm; W = 50 kN = 50×10^3 N;
R_1 = 25 cm; R_2 = 15 cm; P = 2 kW; μ = ?

$$P = \frac{2\pi NT}{60\times1000}\text{kW}$$

$$2 = \frac{2\pi\times150\times T}{60\times1000}$$

$\therefore$ Frictional torque, T = 127.324 Nm = 12732.4 N.cm

But,
$$T = \frac{2}{3}\mu W\frac{\left(R_1^3 - R_2^3\right)}{\left(R_1^2 - R_2^2\right)}$$

$$12732.4 = \frac{2}{3}\times\mu\times\left(50\times10^3\right)\times\frac{\left(25^3 - 15^3\right)}{\left(25^2 - 15^2\right)}$$

$$\mu = 0.0124$$

4. (f) **(i) Free vibration**

"The vibrations of a system because of its own elastic properties. No external exciting force acts in this case."

(ii) Forced vibration

"The vibrations which the system executes under an external periodic force". The frequency of vibrations in this case is same as that of execution of excitation.

(iii) Natural frequency

'Frequency of free vibration of the system'. It is constant for a given system and is represented by 'f_n'

(iv) Degree of freedom

The number of independent co-ordinates required to describe the motion of a system is called degrees of freedom of the system.

5. (a) **Causes of Vibrations**

1. Unbalanced forces and couples existing in the machine itself.
2. Metal to metal contact, which is the dry friction between the two mating surfaces.
3. External excitations acting on the system. These excitations may be periodic, random or impact type.
4. Earthquakes : Many buildings, dams, civil structures etc. get damaged due to this.
5. Winds : Mainly, it effects on the transmission and telephone lines under certain conditions.

Advantages of Vibrations

1. Musical instruments.
2. Vibration testing equipment.
3. Vibratory conveyors.
4. Vibratory compactors.
5. Vibratory screens.
6. Vibratory shakers.
7. For stress relieving purposes.

Disadvantages of Vibrations

1. It creates high repeated stresses.
2. It creates undesirable noise.
3. It has a bad psychological effect on people in the vicinity, is tiring, slows production and creates a generally undesirable condition.
4. Loosseness of parts in the assembled unit.
5. Partial or complete failure of parts etc.

5. (b) For all the cases draw the configuration diagram on a suitable scale in which OA = 10 cm, AP = 40 cm.

(a) When the crank is at i.d.c. ($\theta = 0°$) : Refer Figure 1.

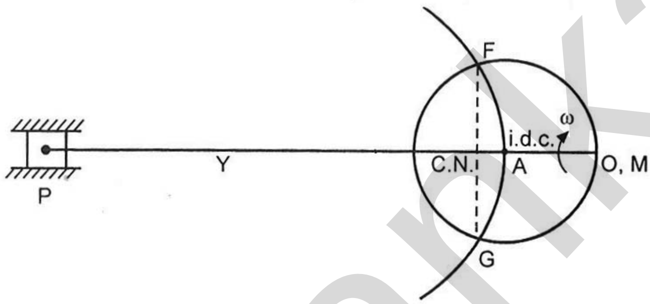


Fig. 1

Velocity diagram : Extension of link PA and perpendicular to OP at O will cut at M. Therefore, Δ OAM is velocity diagram. Velocity of piston.

$$V_p = \omega.(OM) = 0$$

Angular velocity of connecting rod,

$$\omega_c = \frac{V_{PA}}{PA} = \frac{\omega.AM}{PA} = \frac{20 \times 10}{40} = 5 \text{ rad/s}$$

Acceleration diagram : draw a circle A as centre. AM as radius and another circle PA as diameter. Intersection points of two circles are F and G. Join the points F and G and it cuts the connecting rod PA at C and stroke OP at N.

$$f_p = \omega^2 . ON = (20)^2 \times 13 = 5200 \text{ cm/s}^2$$
$$= 52 \text{ m/s}^2$$

$$\alpha_{Pa} = \frac{f^t_{PA}}{PA} = \frac{\omega^2 \cdot CN}{PA} = 0$$

(b) At $\theta = 45°$

The velocity and acceleration diagrams have been drawn as shown in Figure 2.

Velocity of piston :

$$V_P = \omega(OM) = 20\ (6.8) = 136 \text{ cm/s}$$

Angular velocity of connecting rod,

$$\omega_C = \frac{V_{PA}}{PA} = \frac{w \cdot AM}{PA} = \frac{20 \times 3.4}{40} = 1.7 \text{ rad/s}$$

Acceleration of piston,

$f_p = \omega^2(ON) = (20)^2\ (3.8) = 1420\ cm/s^2$

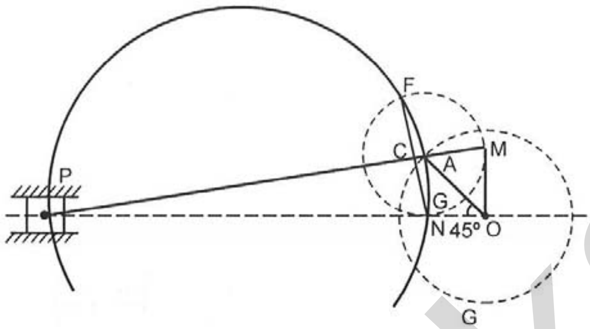


Fig. 2

Angular acceleration of connecting rod,

$$\omega_c = \frac{f_{PA}^t}{PA} = \frac{\omega^2.CN}{PA} = \frac{(20)^2 \times 5.4}{40} = 54\ rad/s^2$$

(c) At θ = 90°

The Klein's construction at θ = 90° had been shown in Figure 3.

Velocity diagram :

The lines drawn as perpendiculars to OP at O and extension of link PA cut the previous line at M which coincides with point A.

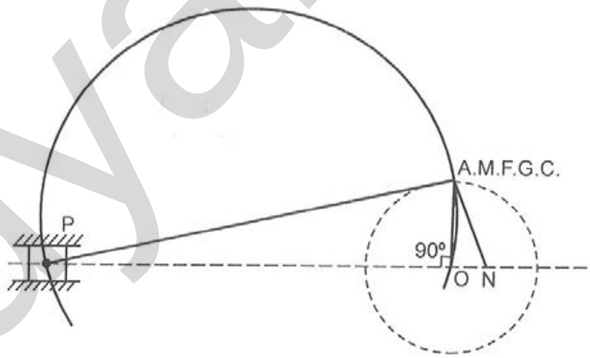


Fig. 3

$$V_P = \omega.OM = 20 \times 8 = 160\ cm/s$$

$$\omega_c = \frac{V_{PA}}{PA} = \frac{\omega \cdot AM}{PA} = 0(\text{since, } AM = 0)$$

Acceleration diagram :

The circle drawn with centre A and AM as radius is a point circle. Another circle is drawn with PA as diameter which shall cut the previous point circle at F and G. These points coincide with A. In order to get the point N, draw the tangent to the circle with PA as diameter which cuts the line of stroke at N. Then quadriangle OACN represents the acceleration diagram.

$f_p = (\omega^2)\,ON = (20)^2 \times 2.8$
$= -1120$ cm/s^2

$$\alpha_c = \frac{f^t_{PA}}{PA} = \frac{\omega^2 \cdot CN}{PA} = \frac{(20)^2 \times 8.4}{40} = 84 \text{ rad/s}^2$$

(d) At θ = 135°

The Klein's construction has been shown in Figure 4

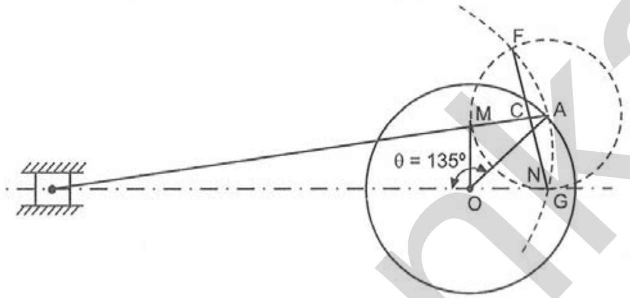


Fig. 4

$V_P = \omega.OM = 20 \times 4.8$
$= 96$ cm/s

$$\omega_C = \frac{V_{PA}}{PA} = \frac{\omega.AM}{PA} = \frac{20 \times 5.6}{40}$$
$= 2.8$ rad/s

$f_p = \omega^2.ON = (20)^2 \times 5.2$
$= 2080$ cm/s^2

$$\alpha_C = \frac{f^t_{PA}}{PA} = \frac{\omega^2.CN}{PA} = \frac{(20)^2 \times 5.4}{40}$$
$= 54$ rad/s^2

5. (c) Draw the configuration diagram as shown in Figure 1 on suitable scale.

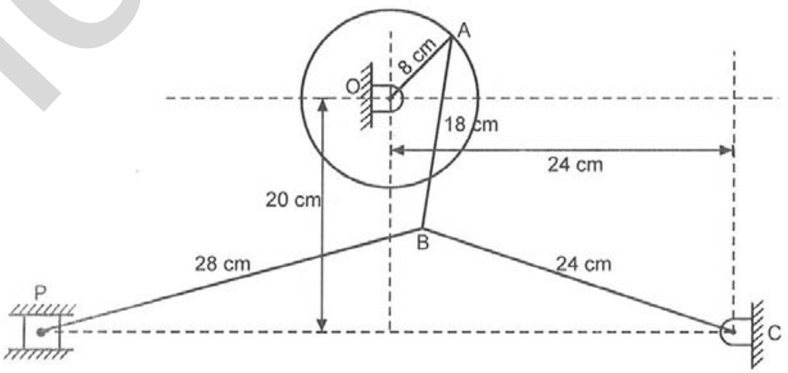


Fig. 1

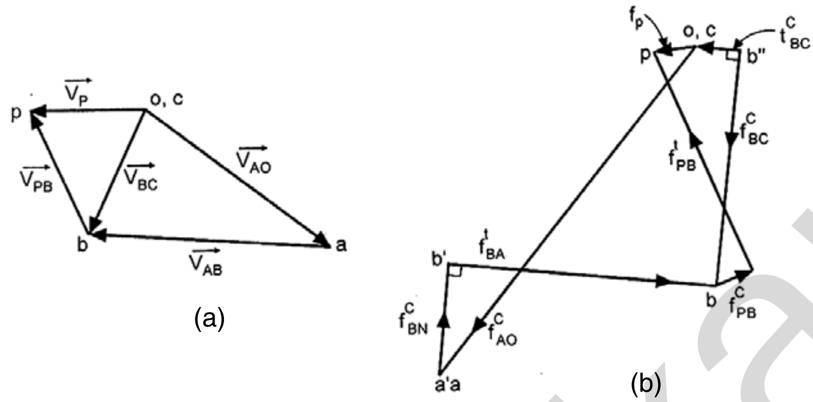


Fig. 2

$$\overrightarrow{V_{AO}} = \overrightarrow{oa} = \frac{2\pi N}{60} \times OA$$

$$= \frac{2\pi \times 105}{60} \times 8 = 88 \text{ cm/s}$$

Draw velocity diagram as shown in Figure 2(a).

(i) Draw $\overrightarrow{oa}$ perpendicular to OA to represent $\overrightarrow{V_{AO}}$ on some suitable scale equal to 88 cm/s.

(ii) Draw lines to represent $\overrightarrow{V_{BA}} = \overrightarrow{ab}$, perpendicular to link AB and $\overrightarrow{V_{BC}} = \overrightarrow{cb}$ perpendicular to BC. Intersection of these lines is at ‘b’.

(iii) From point b draw $\overrightarrow{V_{PB}} = \overrightarrow{bp}$, perpendicular to BP and from point c draw $\overrightarrow{V_p} = \overrightarrow{cp}$ or $\overrightarrow{oc}$ in horizontal direction of motion of slider P. The intersection of these drawn lines gives point p.

By measurement :

$$\overrightarrow{V_p} = \overrightarrow{op} = 30 \text{ cm/s}$$

Acceleration diagram

Calculate the centripetal component of acceleration of various links as per the following table with the help of velocity diagram as follows :

Link	Length of the link, cm	Velocity V cm/s	$f^c = \frac{V^2}{\text{length}} \times \frac{1}{100} (m/s^2)$
AO	8	88	9.68
BA	18	80	3.55
BC	24	48	0.96
PB	28	47	0.79

And, $\overrightarrow{f^t_{AO}}$ = zero, since the link rotates at a uniform angular speed.

The acceleration diagram can now be drawn as shown in Figure 2(b).

(i) Draw $\overrightarrow{Oa'}$, parallel to OA to represent $\overrightarrow{f^c_{AO}}$ = 9.68 m/s^2 on some suitable scale and $f^t_{AO} = \overrightarrow{a'a} = 0$.

(ii) From 'a' draw $\overrightarrow{ab'}$ parallel to link BA equal to 3.55 m/s^2 and directed towards the centre A to represent $\overrightarrow{f^c_{BA}}$ and draw a line perpendicular to link BA to represent $\overrightarrow{f^t_{BA}}$ by vector $\overrightarrow{b'b}$ at later stage. From c draw vector $\overrightarrow{cb''}$ parallel to link BC equal to 0.96 m/s^2 and directed towards the centre C to represent $\overrightarrow{f^c_{BC}}$ and draw a line perpendicular to link BC from point b″ to represent $\overrightarrow{f^t_{BC}}$. Intersection of these two lines plots the point b.

(iii) From point b draw $\overrightarrow{bp'}$ to represent $\overrightarrow{f^c_{PB}}$ equal to 0.79 m/s^2 and draw a line from p′ in the direction perpendicular to link PB to represent $\overrightarrow{f^t_{PB}}$. Also, draw another line from point 'o' or 'c' in horizontal direction to represent $\overrightarrow{f_p}$. Intersection of these two lines plots the point p.

By measurement :

$$\overrightarrow{f_p} = \overrightarrow{op} = 0.85 \text{ m/s}^2$$

6. (a) Given data :

Type of follower = Roller follower

Angle of outstroke (θ_o) = 120° (with S.H.M.)

Angle of dwell after the outstroke (θ_{d_1}) = 30°

Angle of return stroke (θ_r) = 150° (with uniform acceleration and retardation)

Angle of dwell after return stroke $(\theta_{d_2}) = 360° - (\theta_o + \theta_{d_1} + \theta_r)$

$$\theta_{d_2} = 60°$$

Lift of follower = S = 3 cm

Minimum radius of cam (r_c) = 3.5 cm

Radius of roller (r_r) = $\dfrac{1.5}{2}$ = 0.75 cm

For displacement diagram :

Scale : On θ-axis : 1 cm = 30°

On S-axis : 1 cm = 1 cm

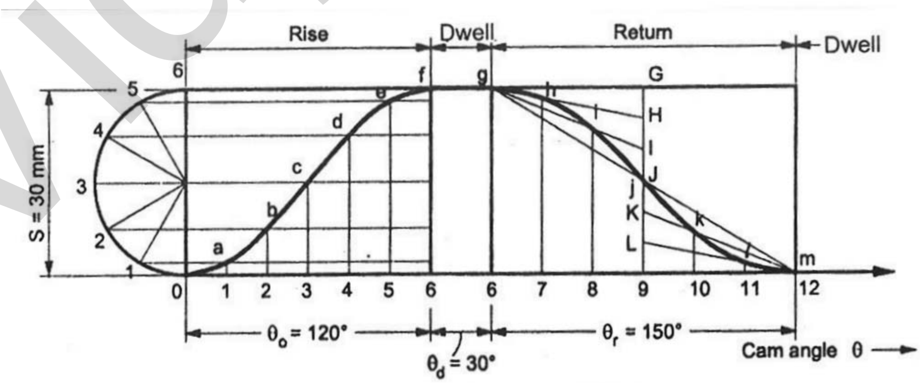


Fig. 1 : Displacement Diagram

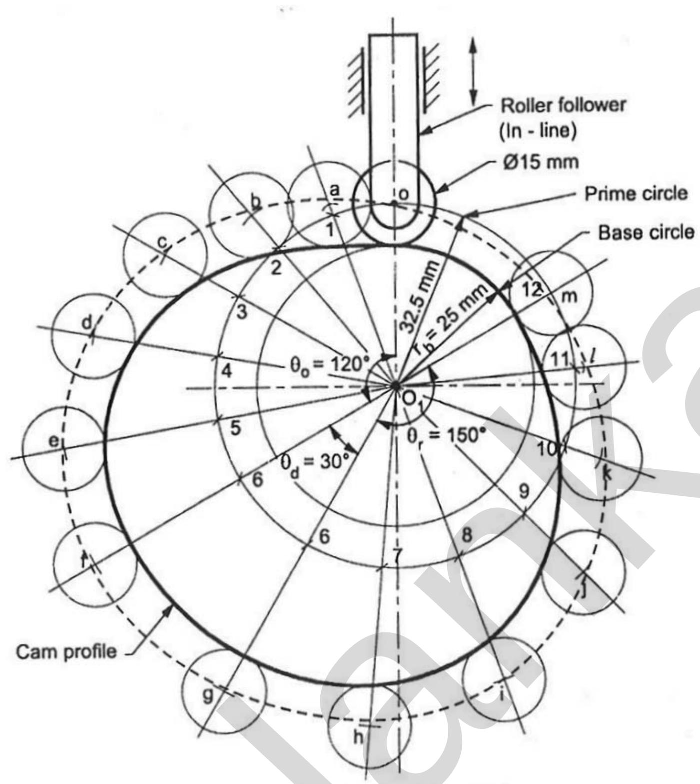


Fig. 2 : Cam Profile

6. (b) Condition for Maximum Power Transmission by a Belt

We know that,

$$P = \frac{(T_1 - T_2)V}{1000} \quad \text{and} \quad \frac{T_1}{T_2} = e^{\mu\theta}$$

$$\therefore \quad T_2 = \frac{T_1}{e^{\mu\theta}}$$

$$P = \frac{\left(T_1 - \frac{T_1}{e^{\mu\theta}}\right)V}{1000}$$

$$\therefore \quad P = \frac{T_1\left(1 - \frac{1}{e^{\mu\theta}}\right)}{1000} \times V$$

For a belt drive

$$\left(\frac{1 - \frac{1}{e^{\mu\theta}}}{1000}\right) \text{ is constant} = K$$

$$P = KT_1V$$

But, $T_{max} = T_1 + T_c$

$T_1 = T_{max} - T_c$

$P = K(T_{max} - T_c)\,V$

$T_C = mV^2$

$\therefore\quad P = K(T_{max} - mV^2)V$

$\therefore\quad P = K(T_{max}V - mV^3)$

For maximum power,

$$\frac{dP}{dV} = 0$$

$$\therefore\quad \frac{dP}{dV} = \frac{d}{dV}\left[K\left(T_{max}V - mV^3\right)\right] = 0$$

$\therefore\quad K(T_{max} - 3mV^2) = 0$

$\therefore\quad T_{max} = 3\,mV^2$

$\therefore\quad T_{max} = 3T_c \qquad (\because\ T_c = mc^2)$

$$\therefore\quad T_c = \frac{1}{3}T_{max}$$

6. (c) Given data :

n = 3;	R_2 = 100 mm;	R_1 = 50 mm;
W = 1 kN;	μ = 0.35;	N = 1500 r.p.m;

Now,
$$T = \frac{1}{2}\mu W(R_1 + R_2)\times n$$

$$T = \frac{1}{2}\times 0.35\times 1\times 10^3\,(50+100)\times 3$$

$$T = 78.75\times 10^3\ N$$

Now, power transmitted,

$$P = \frac{2\pi NT}{60\times 1000} = \frac{2\times 3.14\times 1500\times 78.75\times 10^3}{60000}$$

$$P = 12.363\times 10^3\ W$$

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