

**Strength of Materials**

Time: 3 Hrs.]

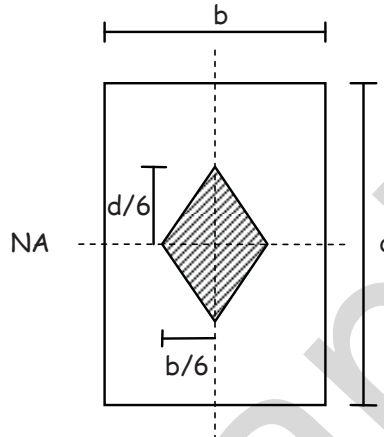
Prelim Question Paper Solution

[Marks : 100

Q.1(a) Attempt any SIX of the following : [12]

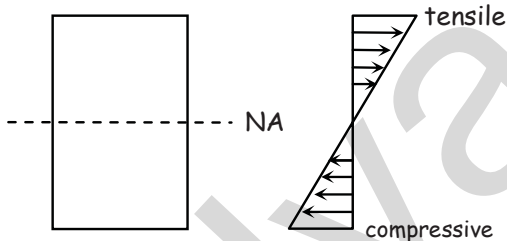
Q.1(a) (i) Draw Core section for rectangular column. [2]

(A) Core of rectangular section,.

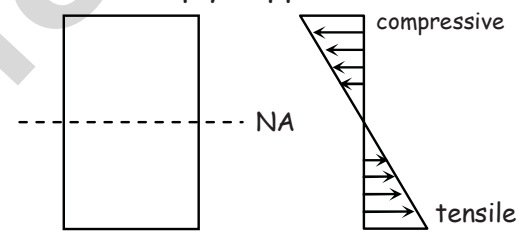


Q.1(a) (ii) Draw stress distribution on Rectangular section subjected to bending. [2]

(A) For cantilever beam



For simply supported



Q.1(a) (iii) Define Poisons Ratio & modular of elasticity. [2]

(A) Poisson's Ratio ( $\mu/v$ )

It is the ratio of lateral strain to longitudinal stress.

$$\mu = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$

**Modulus of Elasticity**

It is a ratio of stress induced in body to the strain.

$$E = \frac{\text{Stress}}{\text{Strain}}$$

Q.1(a) (iv) Define creep. Give one example [2]

(A) **Creep** : Many structural members and machine parts sustain steady loads for long periods of time. For example, beams in a R.C.C. building, plastic mountings for the parts of electronic devices, blades of turbine rotor, etc. Under such conditions, the material may continue to deform and will ultimately

break. Creep continues as long as the load is applied. Therefore, it is a time dependent phenomenon. The greater the time, the more will be the creep. The continuous deformation with time which the material undergoes due to application of external steady loads is called creep or time yield or plastic flow.

**Q.1(a) (v) State any four assumptions in the theory of simple bending. [2]**

**(A)** The following assumptions are made in the theory of simple bending while deriving the flexural formula.

- 1) The material of the beam is homogeneous and isotropic. i.e. the beam is made up of the same material throughout and it has the same elastic properties in all the directions.
- 2) The beam is straight before loading and is of uniform cross-section throughout.
- 3) The beam material is stressed within its elastic limit and thus obeys Hooke's law.
- 4) The transverse sections which were plane before bending remain plane after bending.
- 5) The beam is subjected to pure bending i.e. the effect of shear stresses is totally neglected.
- 6) Each layer of the beam is free to expand or contract independently of the layer above or below it.
- 7) Young's modulus  $E$  for the beam material has the same value in tension and compression.

**Q.1(a) (vi) Give the relationship between  $E$ ,  $G$  and  $K$ . [2]**

**(A)**  $E = 3k(1 - 2\mu)$  where as  
 $E = 2G(1 + \mu)$   $E = \text{Young's modulus}$   
 $K = \text{Bulk modulus}$   
 $G = \text{Shear modulus}$   
 $\mu = \text{Poisson's ratio}$

**Q.1(b) Attempt any TWO of the following : [8]**

**Q.1(b) (i) A load of 5 KN is to be raised with the help of a steel wire. [4]**

**Find the minimum diameter of the steel wire, if the stress is not to exceed 100 MPa.**

**(A)**  $P = 5 \times 10^3 \text{ N}$ ,  $\sigma = 100 \text{ N/mm}^2$

$$\sigma = \frac{P}{A}$$

$$100 = \frac{5 \times 10^3}{(\pi/4)d^2}$$

$$\therefore d = 7.97 \text{ mm}$$

$$\approx 8 \text{ mm}$$

Q.1(b) (ii) Calculate polar M.I. of a square section having 200 mm as side. [4]

(A) Data: A square of side 200 mm.

To find:  $I_p$

Concept: (i)  $I_p = I_{xx} + I_{yy}$

(ii) For a square section

$$I_{xx} = I_{yy} = \frac{a^4}{12}$$

(i) For a square of side a,

$$I_{xx} = \frac{a \cdot a^3}{12} = \frac{a^4}{12} = \frac{(200)^4}{12} = 1.33 \times 10^8 \text{ mm}^4$$

(ii) For a square section

$$I_{yy} = I_{xx} = 1.33 \times 10^8 \text{ mm}^4$$

$$\therefore I_p = I_{xx} + I_{yy} = 1.33 \times 10^8 + 1.33 \times 10^8 = 2.66 \times 10^8 \text{ mm}^4$$

$$I_p = 2.66 \times 10^8 \text{ mm}^4$$

Q.1(b) (iii) A mild steel flat 150 mm wide by 20 mm thick, 6 m long, carries [4] an axial pull of 300 kN, if the modulus of elasticity of steel is 200 kN/mm<sup>2</sup> and Poisson's ratio = 0.25. Calculate the change in length, width, thickness volume of the flat.

(A) To Find Change in length ( $\delta L$ )

$$6 = \frac{P}{A} = \frac{P}{P \times D} = \frac{300}{150 \times 20} = 0.1 \text{ kN/mm}^2$$

$$e = \frac{6}{E} = \frac{0.1}{200} = 5 \times 10^{-4}$$

$$\text{But } e = \frac{\delta L}{L} \therefore \delta L = e \times L \quad \therefore \delta L = 5 \times 10^{-4} \times 6000$$

$$\delta L = 3 \text{ mm (increase)}$$

To find change in width ( $\delta b$ ) and thickness ( $\delta t$ )

$$\text{Lateral Strain } (eL) = \text{i.e.} = 0.25 \times 5 \times 10^{-4} = 1.25 \times 10^{-4}$$

$$\text{But Lateral Strain} = \frac{\delta b}{b} = \frac{\delta t}{t} = 1.25 \times 10^{-4}$$

$$\therefore \delta b = 1.25 \times 10^{-4} \times 150 = 0.01875 \text{ (decrease)}$$

$$\delta t = 1.25 \times 10^{-4} \times 20 = 2.5 \times 10^{-3} \text{ (decrease)}$$

To Find change in Volume ( $\delta v$ )

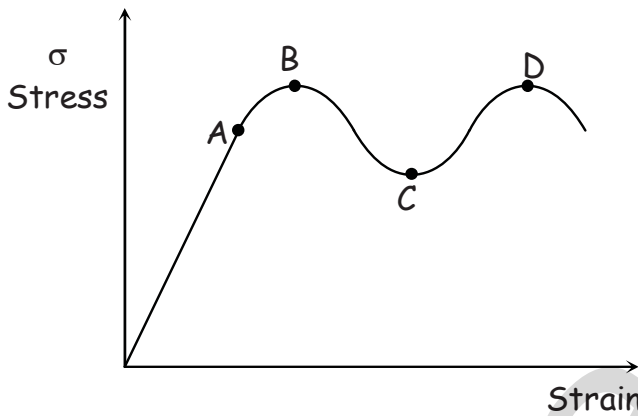
We know  $\frac{\delta v}{v} = e(1 - 2u)$

$\therefore \delta v = e(1 - 2u)v$   
 $= s \times 10^{-4} (1 - 2 \times 0.25) (6000 \times 150 \times 20)$   
 $\therefore \delta v = 4500 \text{mm}^2$

Q.2 Attempt any FOUR of the following : [16]

Q.2(a) Draw the stress strain curve for ductile material and explain the term ultimate stress. [4]

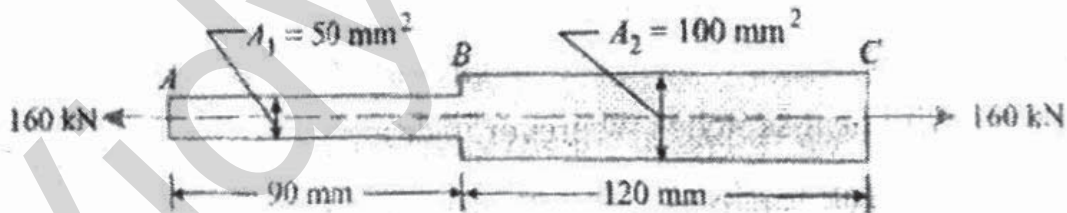
(A)



- A : Elastic limit
- B : Upper Yield stress
- C : Lower Yield stress
- D : Ultimate stress

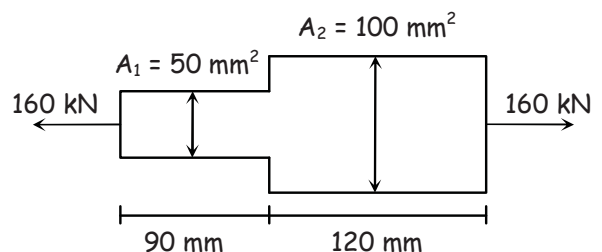
**Ultimate Stress** : It is maximum value of stress the material can withstand before breaking or failure.

Q.2(b) An automobile component shown in fig no. 1. is subjected to a tensile load of 160 kN. Determine the total elongation of the component, if its modulus of elasticity is 200 GPa. [4]



(A)

Total elongation,  $\delta L = \delta L_1 + \delta L_2$   
 $= \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2}$   
 $= \frac{P}{E} \left[ \frac{L_1}{A_1} + \frac{L_2}{A_2} \right]$   
 $= \frac{160 \times 10^3}{2 \times 10^5} \left[ \frac{90}{50} + \frac{120}{100} \right]$   
 $\delta L = 2.4 \text{ mm}$



Q.2(c) A steel rod 4 m long and 20 mm diameter is subjected to an axial [4] tensile load of 45 kN. Find the change in length and diameter of the rod.  $E_s = 2 \times 10^5 \text{ N/mm}^2$ . Poisson's Ratio =  $\frac{1}{4}$ .

(A) Data : Steel rod,  $L = 4 \text{ m} = 4000 \text{ mm}$   
 $d = 20 \text{ mm}$   
 $P = 45 \text{ kN} = 45 \times 10^3 \text{ N}$   
 $E = 2 \times 10^5 \text{ N/mm}^2$   
 $\mu = \frac{1}{4} = 0.25$

To find : (i)  $\delta L$   
(ii)  $\delta d$

Concept : (i)  $\delta L = \frac{PL}{AE}$

(ii) Lateral strain =  $\mu e = \frac{\delta d}{d}$

Solution :  $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (20)^2 = 314.16 \text{ mm}^2$

$\delta L = \frac{PL}{AE} = \frac{(45 \times 10^3) \times 4000}{314.16 \times (2 \times 10^5)}$   
 $= 2.86 \text{ mm (increase)}$

**Note** : Since the rod is subjected to an axial tensile load, there will be increase in the length of the bar and decrease in the diameter.

Linear strain,  $e = \frac{\delta L}{L} = \frac{2.86}{4000} = 0.000715$

Lateral strain =  $\mu \times e = 0.25 \times 0.000715$   
 $= 0.00017875$

$\therefore \frac{\delta d}{d} = 0.00017875 \quad \left( \because \text{lateral strain} = \frac{\delta d}{d} \right)$

$\therefore \frac{\delta d}{20} = 0.00017875$

$\therefore \delta d = 20 \times 0.00017875$   
 $= 0.003575 \text{ mm (decrease)}$

(i)  $\delta L = 2.86 \text{ mm (increase)}$ ,

(ii)  $\delta d = 0.003575 \text{ mm (decrease)}$

Q.2(d) A rod 300 mm long and 20 mm in diameter is heated through  $100^{\circ}\text{C}$  [4] and at the same time pulled by a force 'P'. If the total extension is 0.4 mm. What is the magnitude of 'P'?

Take  $E = 2 \times 10^5 \text{ N/mm}^2$  and  $\alpha = 12 \times 10^{-6} / ^{\circ}\text{C}$ .

(A) Given data

$$L = 300$$

$$d = 20 \text{ mm}$$

$$t = 100^{\circ}$$

$$\text{Total extension} = 0.4 \text{ mm}$$

$$E = 2 \times 10^5 \text{ N/mm}^2 \quad \text{and} \quad \alpha = 12 \times 10^{-6} / ^{\circ}\text{C}$$

Total extension of the rod

= free expansion + extension  $\delta L$  due to force 'P'

$$0.4 = (L \alpha t) + \delta L \text{ due to force 'P'}$$

$$0.4 = (300 \times 12 \times 10^{-6} \times 100) + \delta L$$

$$0.4 = 0.36 + \delta L$$

$$\therefore \delta L = 0.04 \text{ mm}$$

But

$$\delta L = \frac{PL}{AE}$$

$$0.04 = \frac{P \times 300}{\left(\frac{\pi \times 20^2}{4}\right) \times 2 \times 10^5}$$

$$\therefore P = \frac{0.04 \times \left(\frac{\pi}{4} \times 20^2\right) \times 2 \times 10^5}{300}$$

$$\therefore P = 8.377 \times 10^3 \text{ N} \quad \text{or} \quad P = 8.377 \text{ KN}$$

Q.2(e) A cylindrical shell is 8 m long, 1 m internal diameter and 15 mm [4] metal thickness. Calculate circumferential strain and longitudinal strain, if cylindrical shell is subjected to internal pressure of  $1.5 \text{ N/mm}^2$ .

Take  $E = 2 \times 10^5 \text{ N/mm}^2$  and  $\mu = 0.25$ .

(A) Given data :  $L = 8 \text{ m} = 8000 \text{ mm}$   $d = 1 \text{ m} = 1000 \text{ mm}$   $t = 15 \text{ mm}$ ,  $p = 1.5 \text{ N/mm}^2$   
 $E = 2 \times 10^5 \text{ N/mm}^2$   $\mu = 0.25$ .

i) To find circumferential strain ( $e_c$ )

$$e_c = \frac{Pd}{4tE} (2 - \mu) = \frac{1.5 \times 1000}{4 \times 15 \times 2 \times 10^5} (2 - 0.25)$$

$$e_c = 2.1875 \times 10^{-4}$$

ii) To find longitudinal strain ( $e_L$ )

$$e_L = \frac{Pd}{4tE}(1-2\mu)$$

$$= \frac{1.5 \times 1000}{4 \times 15 \times 2 \times 10^5}(1-2 \times 0.25)$$

$$e_L = 6.25 \times 10^{-5}$$

**Q.2(f) A steel bar of 30 mm diameter is heated to 70°C and then clamped [4] at ends. It is then allowed to cool down to 20°C. Calculate temperature stresses developed and reactions at the clamps, length of bar = 10 m,  $\alpha = 12 \times 10^{-6}/^\circ\text{C}$ ;  $E = 2 \times 10^5 \text{ N/mm}^2$ .**

**(A)** Diameter  $d = 30 \text{ mm}$   
 Fall in temperature  $t = 70 - 20$   
 $= 50^\circ\text{C}$

Length of bar  $L = 10 \text{ m}$   
 $= 10000 \text{ mm}$

$\alpha = 12 \times 10^{-6}/^\circ\text{C}$ ,  $E = 2 \times 10^5 \text{ N/mm}^2$

$\therefore$  Temperature Stress

$$\sigma = \alpha t E$$

$$= 12 \times 10^{-6} \times 50 \times 2 \times 10^5$$

$$= 120 \text{ N/mm}^2 \text{ (Tensile)}$$

Reactions at the clamps

$$B = 6A$$

$$= \alpha t E \times \left( \frac{\pi d^2}{4} \right) = 120 \times \frac{\pi (30)^2}{4}$$

$$= 84823 \quad = 84.823 \text{ kN}$$

**Q.3 Attempt any FOUR of the following : [16]**

**Q.3(a) A beam AB 10 m long has supports at its ends A and B. It carries a point load of 5 kN at 3 meters from A and a point load of 5 kN at 7 meters from A and a udl of 1 kN per meter between the point loads. Draw S.F. Diagram and B.M. diagram for the beam.**

**(A)**  $\sum F_y = 0$   
 $R_A + R_D = 5 + 5 + (1 \times 4) = 14$   
 $M_A = 0$   
 $R_D \times 10 = (5 \times 1) + (1 \times 4 \times (3 + 2)) + (5 \times 7)$   
 $R_D = 7 = R_A$

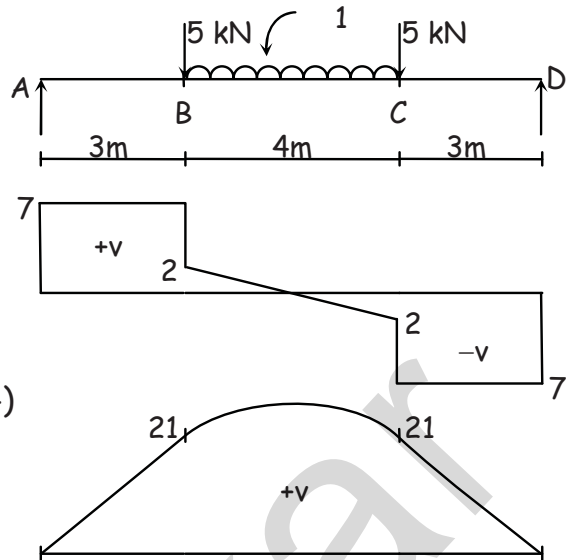
For SFD

$$SF_{AL} = 0, SF_{AR} = 7$$

$$SF_{BL} = 7, SF_{BR} = 7 - 5 = 2$$

$$SF_{CL} = 2 - (1 \times 4) = -2, SF_{DL} = -7$$

$$SF_{CR} = -2 - 5 = -7, SF_{DR} = 0$$



For BMD

$$BM_B = (-1 \times 4 \times 2) + (7 \times 7) - (5 \times 4)$$

$$= 21$$

$$BM_L = 7 \times 3 = 21$$

Q.3(b) A simply supported right side overhanging beam supported at 4 meter and right side 1 meter overhang. A Loaded by udl 10 KN/m over entire span. Draw S.F. diagram and B.M. diagram.

(A)

$$\sum F_y = 0$$

$$R_A + R_B = 10 \times 5 = 50$$

$$M_A = 0$$

$$4R_B = 10 \times 5 \times 2.5$$

$$R_B = 31.25$$

$$R_A = 18.75$$

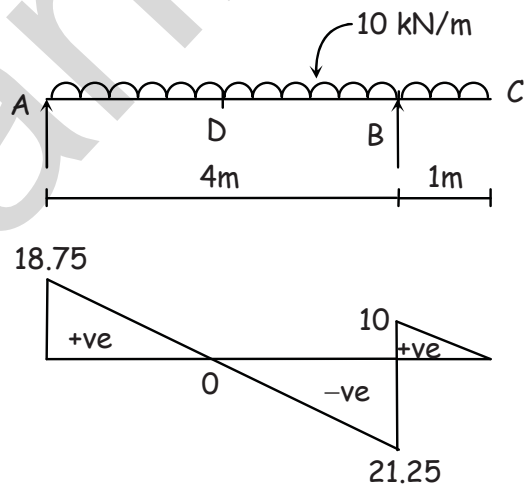
For SFD

$$SF_{AL} = 0 \quad SF_{BL} = 18.75 - 10 \times 4$$

$$SF_{AR} = 18.75 \quad = -21.25$$

$$SF_{CL} = 10 - 10 \quad SF_{BR} = -21.25 + 31.25$$

$$SF_{CR} = 0 \quad = 10$$



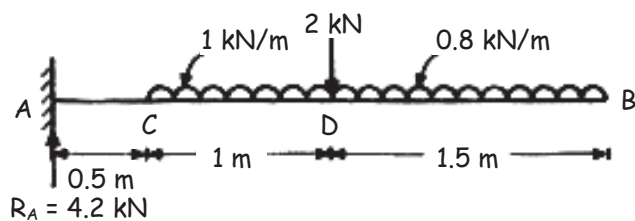
For BMD

$$BM_A = 0$$

$$BM_D = -10 \times 3 \times 1.5 + 31.25 \times 2 = 17.5$$

$$BM_B = -10 \times 1 \times 0.5 = -5$$

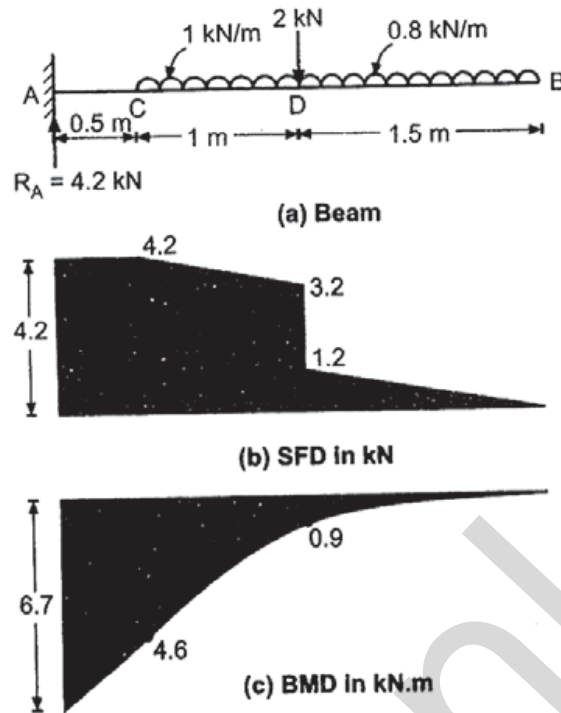
Q.3(c) Draw S.F. and B.M. diagram for the beam as shown in Figure.



[4]

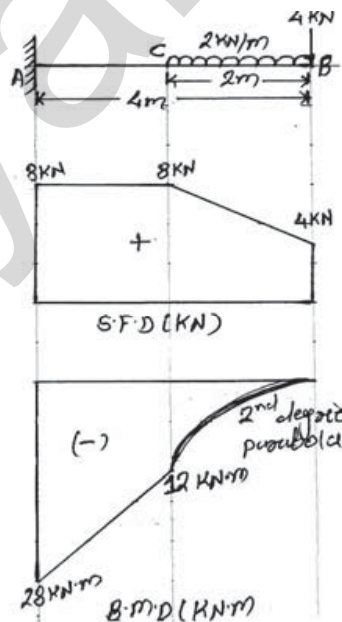


(A)



Q.3(d) A cantilever beam 4 m long carries a udl of 2kN/m over 2 m from [4] free end and a point load of 4kN at free end. Draw S.F. and B.M. diagrams.

(A)



i) support reactions

a)  $\Sigma F_y = 0$ ;  $R_A = 4 + (2 \times 2) = 8\text{ kN}$

[1 mark]

ii) S.F calculation

S.F at just left of A = 0 kN

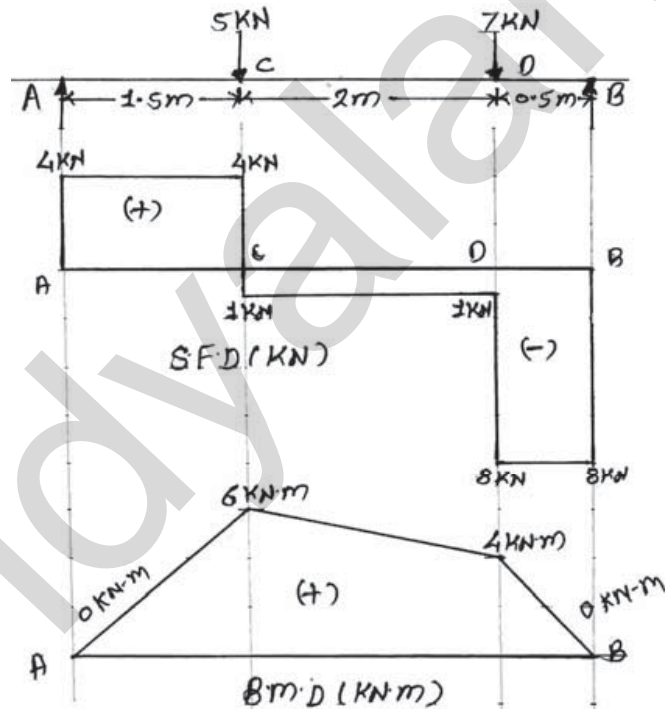
S·F at just right of A =  $R_A = 8\text{KN}$   
 S·F just C =  $8\text{KN}$   
 S·F just left of B =  $8 - (2 \times 2) = 4\text{KN}$   
 S·F just right of B =  $4 - 4 = 0\text{KN}$ .

iii) B·M calculation

B·Mat B = 0...free end  
 B·Mat c =  $-4 \times 2 - 2 \times 2 \times 1$   
 $= -12\text{KN}\cdot\text{M}$   
 B·Mat A =  $-4 \times 4 - 2 \times 2 \times 3$   
 $= -12\text{KN}\cdot\text{m}$   
 B·Mat A =  $-4 \times 4 - 2 \times 2 \times 3$   
 $= -28\text{KNM}$

Q.3(e) A simply supported beam of span 4 m carries two point loads of 5kN [4] and 7kN at 1.5 m and 3.5 m from the left hand support respectively. Draw SFD and BMD showing important values.

(A)



i) Support reactions

a)  $\sum F_y = 0$

$$R_A + R_B = 5 + 7 = 12\text{KN}$$

b)  $\sum m@A = 0$

$$(5 \times 1.5) + (7 \times 3.5) - 4R_B = 0$$

$$32 = 4R_B$$

$$\therefore R_B = 8\text{KN}$$

$$\therefore R_A = 12 - 8 = 4\text{KN}$$

ii) S·F calculation

- S·F at just left of A = 0
- S·F at just right of A =  $R_A = 4\text{KN}$
- S·F at just left of C = 4KN
- S·F at just right of C =  $4 - 5 = -1\text{KN}$
- S·F at just left of D =  $-1\text{KN}\cdot\text{M}$
- S·F at just right of D =  $-1 - 7 = -8\text{KN}$
- S·F at just left of B =  $-8\text{KN}$
- S·F at just right of B =  $-8 + R_B = 0\text{KN}$ .

iii) B·M calculation

- B·M at A = B·M at B = 0 ..... simple support
- B·M at C =  $R_A \times 1.5 = 4 \times 1.5 = 6\text{KN}\cdot\text{M}$
- B·M at D =  $R_A \times 3.5 - (5 \times 2) = 4 \times 3.5 - 10 = 4\text{KN}\cdot\text{M}$

Q.4 Attempt any FOUR of the following :

[16]

Q.4(a) Find M.I. about x-x axis of T-section having flange 150 mm × 50 mm and web 150 mm × 50 mm, overall depth 200 mm. [4]

(A) Find out  $I_{xx}$

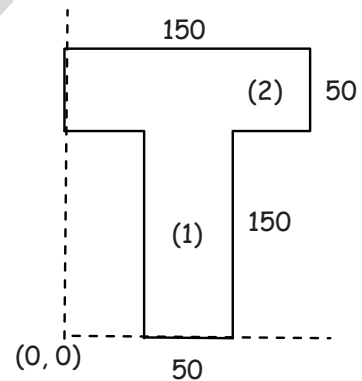
$$A_1 = A_2 = 150 \times 50 = 7500$$

$$y_1 = 75, y_2 = 175$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{7500 \times 75 + 7500 \times 175}{7500 \times 2}$$

$$= \frac{250}{2}$$

$$\therefore \bar{y} = 125$$



$$I_{xx} = I_{xx1} + I_{xx2}$$

$$I_{xx1} = \frac{bd^3}{12} + A(\bar{y} - y_1)^2$$

$$= \frac{50 \times 150^3}{12} + 7500(125 - 75)^2$$

$$= 32.812 \times 10^6 \text{ mm}^4$$

$$I_{xx2} = \frac{bd^3}{12} + A(\bar{y} - y_2)^2$$

$$= \frac{150 \times 50^3}{12} + 7500(125 - 175)^2$$

$$= 20.312 \times 10^6 \text{ mm}^4$$

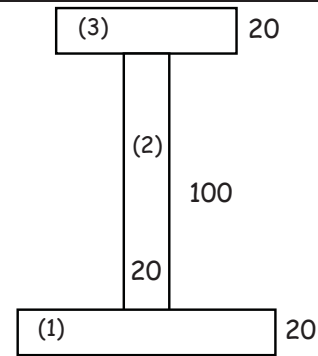
$$I_{xx} = 53.214 \times 10^6 \text{ mm}^4$$

Q.4(b) An I-section have the following dimensions Top flange 60 mm × 20 mm. bottom flange 100 mm × 20 mm, web 100 mm × 20 mm, overall depth 140 mm. Find the M.I. about y-y axis. [4]

(A) To find  $I_{yy}$

$$\therefore x_1 = x_2 = x_3 = \bar{x} = 50$$

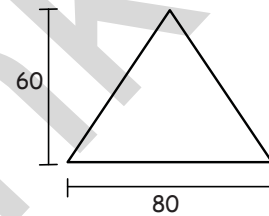
$$\begin{aligned} \therefore I_{yy} &= I_{yy1} + I_{yy2} + I_{yy3} \\ &= \left[ \frac{db^3}{12} \right]_1 + \left[ \frac{db^3}{12} \right]_2 + \left[ \frac{db^3}{12} \right]_3 \\ &= \frac{20 \times 100^3}{12} + \frac{100 \times 20^3}{12} + \frac{20 \times 60^3}{12} \\ I_{yy} &= 2.093 \times 10^6 \text{ mm}^4 \end{aligned}$$



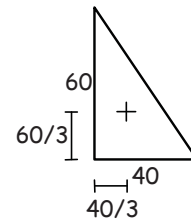
Q.4(c) An isosceles triangular section ABC has a base width 80 mm and height 60 mm. Determine the M.I. of the section about c.g. of the section and the base BC.

(A) (i) MI about CG

$$\begin{aligned} I_{xx} &= \frac{bd^3}{36} \\ &= \frac{80 \times 60^3}{36} \\ I_{xx} &= 0.48 \times 10^6 \end{aligned}$$



$$\begin{aligned} I_{yy} &= 2 \left[ \frac{db^3}{36} + A \left( \frac{40}{6} \right)^2 \right] \\ &= 2 \left[ \frac{60 \times 40^3}{36} + \frac{1}{2} \times 60 \times 40 \times \left( \frac{40}{3} \right)^2 \right] \\ I_{yy} &= 0.64 \times 10^6 \text{ mm}^4 \end{aligned}$$



(ii) MI about BASE

$$\begin{aligned} &= \frac{bd^3}{36} + A \left( \frac{60}{3} \right)^2 = \frac{80 \times 60^3}{36} + \frac{1}{2} \times 60 \times 80 \times \left( \frac{60}{3} \right)^2 \\ &= 19.68 \times 10^6 \text{ mm}^4 \end{aligned}$$

Q.4(d) Calculate moment of inertia of a hollow rectangle about an axis [4] passing through base 200 mm size, it has the following details.

(i) internal dimension = 160 mm × 260 mm

(ii) external dimension = 200 mm × 300 mm

(A) Data : A hollow rectangle as shown in Fig.

$$b = 160 \text{ mm}, d = 260 \text{ mm},$$

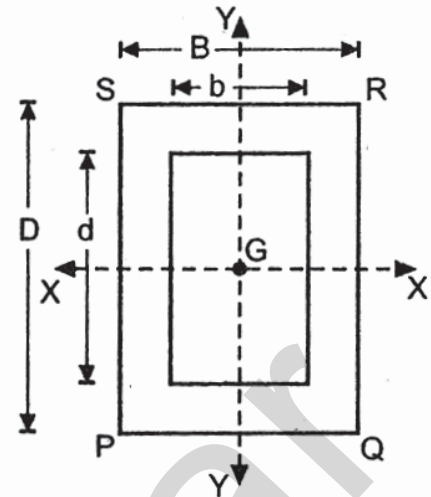
$$B = 200 \text{ mm}, D = 300 \text{ mm}$$

To find :  $I_{\text{Base}}$ .

Concept : (i) M.I. of hollow rectangle.

(ii) Use of parallel axis theorem.

We have to calculate the moment of inertia of a hollow rectangle about the base PQ. Base PQ is parallel to XX axis.



$$(i) \quad I_{XX} = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{1}{12}(BD^3 - bd^3)$$

$$= \frac{1}{12} [200 \times 300^3 - 160 \times 260^3]$$

$$= 215653333.3 \text{ mm}^4$$

$$(ii) \quad A = BD - bd$$

$$= 200 \times 300 - 160 \times 260$$

$$= 18400 \text{ mm}^2$$

(iii) Distance between base PQ and XX axis,

$$h = \frac{D}{2} = \frac{300}{2} = 150 \text{ mm}$$

(iv) Applying the theorem of parallel axis,

M.I. about parallel axis = M.I. about C.G. axis +  $A \times h^2$

$$I_{PQ} = I_{XX} + Ah^2$$

$$= 215653333.3 + 18400 \times 150^2$$

$$= 435563333.3 \text{ mm}^4$$

$$I_{PQ} = 435563333.3 \text{ mm}^4$$

Q.4(d) A symmetrical I-section has the following dimensions. Calculate [4] Polar M.I. of the section. Flanges = 100 mm × 10 mm, Web = 10 mm × 100 mm.

(A) Data : A symmetrical I section as shown in Fig.

To find :  $I_p$

Concept : Calculate  $I_{XX}$  and  $I_{YY}$ .

$$I_p = I_{XX} + I_{YY}$$

(i) Since the section is symmetrical about XX axis, its M.I. can be determined by considering a hollow rectangular section.

$$B = 100 \text{ mm},$$

$$D = 10 + 100 + 10 = 120 \text{ mm},$$

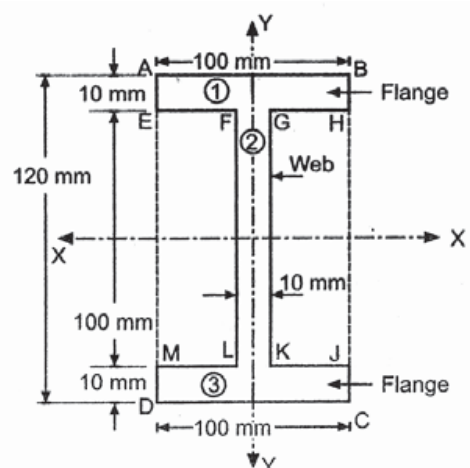
$$b = (100 - 10) = 90 \text{ mm},$$

$$d = 100 \text{ mm}$$

$$(ii) \quad I_{XX} = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{1}{12}(BD^3 - bd^3)$$

$$= \frac{1}{12} [100 \times 120^3 - 90 \times 100^3]$$

$$= 6.9 \times 10^6 \text{ mm}^4$$



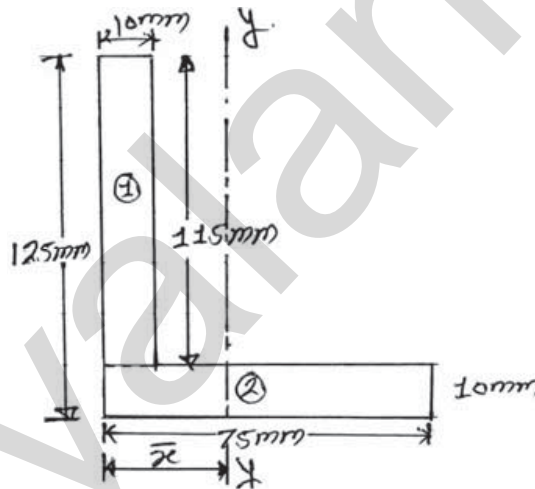
**Note:** M.I. about YY axis should not be calculated as  $\frac{1}{12}(DB^3 - db^3)$  since the c.g. of rectangles EFLM and GHJK are not lying on YY axis.

(iii)  $I_{yy} = 2 \times \text{M.I. of flanges} + \text{M.I. of web}$   
 $= 2 \times \left( \frac{10 \times 100^3}{12} \right) + 100 \times \frac{10^3}{12} = 1.675 \times 10^6 \text{ mm}^4$

(iv) Polar M.I. of section,  
 $I_p = I_{zz} = I_{xx} + I_{yy} = 6.9 \times 10^6 + 1.675 \times 10^6$   
 $= 8.575 \times 10^6 \text{ mm}^4$   
 $I_p = 8.575 \times 10^6 \text{ mm}^4$

**Q.4(e)** Find  $I_{yy}$  for an unequal angle section having vertical leg of  $125 \times 10$  [4] mm and horizontal leg of  $75 \times 10$  mm.

(A)



i) Position of  $y$ - $y$  axis ( $\bar{x}$ )

$$a_1 = 115 \times 10 = 1150 \text{ mm}^2 \qquad a_2 = 75 \times 10 = 750 \text{ mm}^2$$

$$x_1 = \frac{10}{2} = 5 \text{ mm} \qquad x_2 = \frac{75}{2} = 37.5 \text{ mm}$$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(1150 \times 5) + (750 \times 37.5)}{1150 + 750}$$

$$\bar{x} = 17.828 \text{ mm}$$

ii) M.I. about  $y$ - $y$  is given by parallel axis theorem

$$I_{yy} = I_{yy1} + I_{yy2}$$

$$I_{yy1} = I_{G1} + A_1 h_1^2 = \left[ \frac{db^3}{12} + bd(\bar{x} - x_1)^2 \right]_{(1)}$$

$$I_{yy1} = \frac{115 \times 10^3}{12} + 1150(17.828 - 5)^2 = 198.82 \times 10^3 \text{ mm}^4$$

$$I_{yy2} = I_{G2} + A_2 h_2^2 = \frac{10 \times 75^3}{12} + 750(17.828 - 37.5)^2$$

$$I_{yy2} = 641.80 \times 10^3 \text{ mm}^4$$

$$\therefore I_{yy} = 840.62 \times 10^3 \text{ mm}^4$$

Q.5 Attempt any FOUR of the following : [16]

Q.5 (a) A rectangular beam 60 mm wide and 150 mm deep is simply supported over a span of 6 m. If the beam is subjected to central point load 12 kN, Find maximum bending stress induced in the beam section.

(A)  $b = 60$ ,  $d = 150$

$$\text{Max. BM} = 6 \times 3 = 18 \text{ kNm}$$

$$= 18 \times 10^6 \text{ N-mm}$$

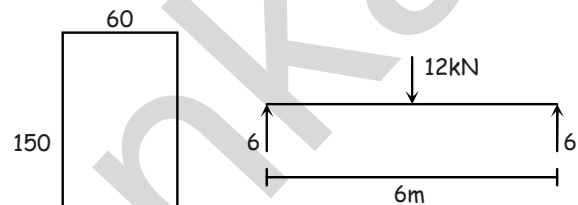
$$I = \frac{60 \times 150^3}{12}$$

$$y = \frac{150}{2} = 75$$

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{MY}{I} = \frac{18 \times 10^6 \times 75}{\frac{60 \times 150^3}{12}} = 80 \text{ N/mm}^2$$

$$\sigma_{\max} = 80 \text{ N/mm}^2$$



Q.5 (b) Calculate the limit of eccentricity for a circular section having diameter 50 mm. [4]

(A) For limit of eccentricity,

$$\sigma_{\text{direct}} = \sigma_{\text{bend}}$$

$$\frac{P}{A} = \frac{MY}{I}$$

$$\frac{P}{(\pi/4)d^2} = \frac{Pe \times (d/2)}{(\pi/64)d^4}$$

$$\therefore e = \frac{d}{8}$$

$$= \frac{50}{8}$$

$$e = 6.25$$

Q.5 (c) A rectangular strut is 150 mm and 120 mm thick. It carries a load [4] of 180 kN at an eccentricity of 10 mm in a plane bisecting the thickness. Find the maximum and minimum intensities of stress in the section.

(A)  $e = 10$ ,  $P = 18 \times 10^4$  N

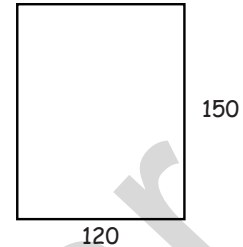
$$\text{Total stress} = \sigma_{\text{direct}} \pm \sigma_{\text{bend}}$$

$$\sigma_{\text{direct}} = \frac{P}{A} = \frac{18 \times 10^4}{120 \times 150} = 10 \text{ (comp)}$$

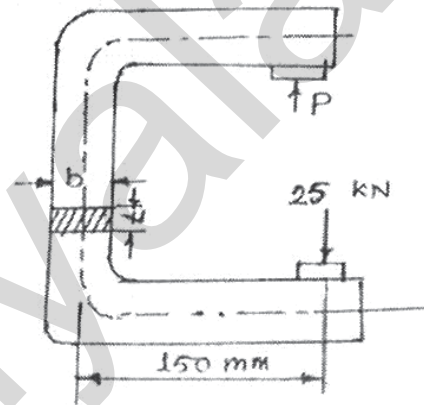
$$\sigma_{\text{bend}} = \frac{MY}{I} = \frac{Pe \times Y}{I} = \frac{18 \times 10^4 \times 10 \times 75}{120 \times \frac{150^3}{12}} = 4$$

$$\therefore \text{Max. stress} = -10 - 4 = -14 \quad \text{i.e. 14 (comp)}$$

$$\text{Min. stress} = -10 + 4 = -6 \quad \text{i.e. 6 (comp)}$$



Q.5 (d) A c-clamp as shown in fig. no 2 carries a load  $P = 25$  kN. The cross [4] section of the clamp at x-x is rectangular, having width equal to twice the thickness. Assuming that the c-clamp is made of steel casing with allowable stress of  $100 \text{ N/mm}^2$ . Find its dimensions.



(A)  $P = 25 \times 10^3$ ,  $\sigma = 100 \text{ N/mm}^2$ ,  $b = 2t$

$$\text{Total stress} = \sigma_{\text{direct}} + \sigma_{\text{bend}}$$

$$\sigma_{\text{direct}} = \frac{P}{A} = \frac{25 \times 10^3}{b \times t} = \frac{25 \times 10^3}{2t^2}$$

$$\sigma_{\text{bend}} = \frac{MY}{I} = \frac{25 \times 10^3 \times 150 \times b / 2}{tb^3 / 12}$$

$$= \frac{25 \times 10^3 \times 150 \times t \times 12}{tb^3} = \frac{25 \times 10^3 \times 150 \times 12}{8t^3}$$

$$100 = \frac{25 \times 10^3 \times 150 \times 12}{8t^3} + \frac{25 \times 10^3}{2t^2}$$

$\therefore$  Solving for point  $t$ ,  $t = 39.18$  mm

$\therefore b = 78.36$  mm



Q.5 (e) Determine the maximum bending stress developed in a beam of [4]  
 rectangular cross-section 50 mm × 150 mm when a bending moment  
 of 600 N.m is applied about X-X axis.

- (A) Data : A rectangular section as shown in Figure 1,  
 $b = 50 \text{ mm}$ ,  $d = 150 \text{ mm}$   
 $M = 600 \text{ N.m} = 600 \times 10^3 \text{ N.mm}$

To find :  $\sigma$

Concept : Use of bending stress equation  $\frac{M}{I} = \frac{\sigma}{y}$ .

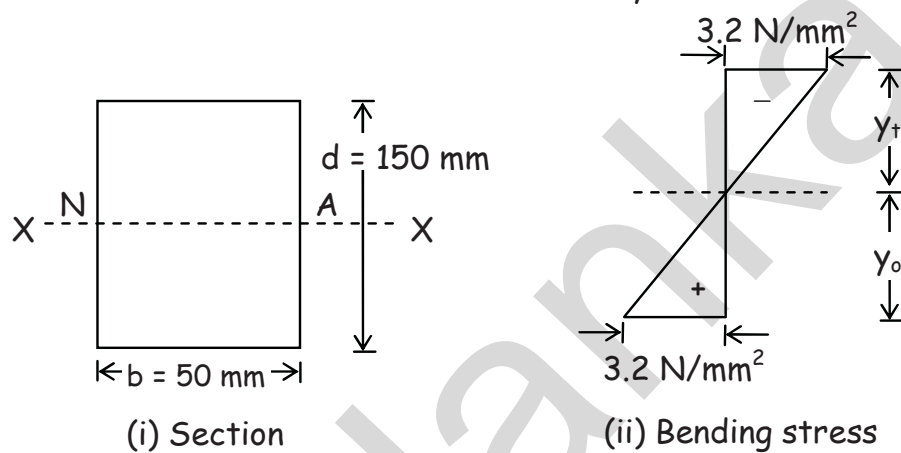


Fig. 1

Step (i) :  $I_{xx} = \frac{bd^3}{12} = \frac{50 \times 150^3}{12} = 14062500 \text{ mm}^4$

Step (ii) :  $y = \frac{d}{2} = \frac{150}{2} = 75 \text{ mm}$

Step (iii) : Using the relation,

$$\frac{M}{I} = \frac{\sigma}{y}$$

$\therefore \frac{600 \times 10^3}{14062500} = \frac{\sigma}{75}$

$$\sigma = \frac{600 \times 10^3 \times 75}{14062500} = 3.2 \text{ N/mm}^2$$

$$\sigma = 3.2 \text{ N/mm}^2$$

Q.6 Attempt any FOUR of the following :

[16]

Q.6(a) State the equation of torsion and write the notations used in it.

[4]

(A)  $\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$

T = torque applied

$J$  = polar MI

$\tau$  = shear stress at radius  $r$

$G$  = shear modulus

$\theta/2$  = angle of twist per unit length

**Q.6(b) A solid circular shaft of 120 mm diameter is transmitting power of [4] 120 KW at 150 rpm. Find the intensity of the shear stress induced in the shaft. Take  $T_{\max} = 1.4 T_{\text{avg}}$ .**

**(A)**  $d = 120$  mm,  $P = 120$  kW,  $N = 150$  rpm,  $T_{\max} = 1.4 T_{\text{avg}}$

$$P = \frac{2\pi NT}{60}$$

$$120 \times 10^3 = \frac{2\pi \times 150 T}{60}$$

$$T = 7.639 \times 10^3 \text{ Nm}$$

$$T_{\text{avg}} = 7.639 \times 10^6 \text{ Nmm}$$

$$\therefore T_{\max} = 10.695 \times 10^6 \text{ Nmm}$$

$$\frac{T_{\max}}{J} = \frac{\tau}{r}$$

$$\therefore \tau = \frac{T_{\max} r}{J} = \frac{10.695 \times 10^6 \times 60}{(\pi/32) 120^4} = 31.521 \text{ N/mm}^2$$

**Q.6(c) Find power transmitted by a shaft having 60 mm diameter rotating [4] at 120 rpm. If maximum permissible shear stress = 80 MPa.**

**(A)**  $d = 60$  mm,  $N = 120$  rpm,  $\tau = 80$

$$T_{\max} = \frac{\tau J}{r}$$

$$T_{\max} = \frac{80 \times \pi \times 60^4}{30 \times 32}$$

$$= 3.392 \times 10^3 \text{ Nmm}$$

$$= 3.392 \times 10^3 \text{ Nm}$$

$$\text{Power} = \frac{2\pi NT}{60}$$

$$= \frac{2\pi \times 120 \times 3.392 \times 10^3}{60}$$

$$= 42.636 \times 10^3 \text{ W}$$

$$\text{Power} = 42.636 \text{ kW}$$

Q.6(d) A shaft of hollow circular cross section has outer diameter 120 mm, [4]  
inner 90 mm. It is subjected to a torsional moment of 18 kNm.  
For this shaft compute shear stress at the outer surface.

(A)  $d_1 = 120 \text{ mm}$ ,  $d_2 = 90 \text{ mm}$ ,  $T = 18 \text{ kNm}$

$$\frac{T}{J} = \frac{\tau}{r}$$

$$\tau = \frac{Tr}{J} = \frac{18 \times 10^6 \times 60}{(\pi/32)(120^4 - 90^4)}$$

$$\tau_{\max} = 77.606 \text{ N/mm}^2$$

Maximum shear stress occurs at outer surface.

Q.6(e) Find the torque that can be applied to a shaft of 100 mm in [4]  
diameter, if the Permissible angle of twist is 2.75 in a length of  
6m. Take  $G = 30 \text{ kN/mm}^2$

(A) Data : Solid shaft,  $D = 100 \text{ mm}$ ,  $\theta = 2.75^\circ = \left(2.75 \times \frac{\pi}{180}\right) \text{ rad}$ ,

$$L = 6 \text{ m} = 6 \times 10^3 \text{ mm}$$

$$C = 80 \text{ kN/mm}^2 = 80 \times 10^3 \text{ N/mm}^2$$

To find : T

Concept : Use of torsional formula.

Using the relation,

$$\frac{T}{I_p} = \frac{C\theta}{L}$$

$$\therefore \frac{T}{\frac{\pi}{32} D^4} = \frac{C\theta}{L}$$

$$\therefore \frac{T}{\frac{\pi}{32} (100)^4} = \frac{(80 \times 10^3) \left(2.75 \times \frac{\pi}{180}\right)}{6 \times 10^3}$$

$$T = 6282734.283 \text{ N.mm}$$

$$= 6282.734 \text{ N.m} = 6.282 \text{ kN.m}$$

Q.6(f) A solid circular shaft of 120 mm diameter is transmitting power of [4]  
100 kW at 150 rpm. Find the intensity of the shear stress induced  
in the shaft.

Take  $T_{\max} = 1.4 T_{\text{avg}}$

(A) Data: Solid shaft,  $D = 120 \text{ mm}$

$$\text{Power } P = 100 \text{ kW} = 100 \times 10^3 \text{ W}$$

$$\text{Speed } N = 150 \text{ rpm,}$$

$$T_{\max} = 1.4 T_{\text{avg}} = 1.4 T_{\text{mean.}}$$

To find :  $f_s$

Concept : (i) Use equation of power first to find  $T_{\text{mean}}$ .

(ii) Find  $T_{\max}$  by knowing  $T_{\text{mean}}$ .

(iii) Use equation of  $T_{\max}$  to find  $f_s$ .

$$(i) \quad \text{Power } P = \left( \frac{2\pi N T_{\text{mean}}}{60} \right) W$$

$$100 \times 10^3 = \frac{2\pi \times 150 \times T_{\text{mean}}}{60}$$

$$T_{\text{mean}} = 6366.197 \text{ N.m}$$

$$(ii) \quad T_{\max} = 1.4 T_{\text{mean}} = 1.4 \times 6366.197$$

$$= 8912.68 \text{ N.m}$$

$$= 8912.68 \times 10^3 \text{ N.mm}$$

$$(iii) \quad T_{\max} = \frac{\pi}{16} f_s D^3$$

$$8912.68 \times 10^3 = \frac{\pi}{16} \times f_s \times (120)^3$$

$$f_s = 26.268 \text{ N/mm}^2$$

Note:  $f_s$  can also be calculated by using the relation,

$$\frac{T}{I_p} = \frac{f_s}{R}$$

$$\frac{8912.68 \times 10^3}{\frac{\pi}{32} (120)^4} = \frac{f_s}{\left( \frac{120}{2} \right)}$$

$$\therefore f_s = 26.268 \text{ N/mm}^2$$

