S.Y. Diploma : Sem. III
[CE/CS/CR/CV]

Mechanics of Structure

Time: 3 Hrs.] Prelim Question Paper Solution **[Marks : 70**

Q.1(a) Attempt any SIX of the following. [12]

Q.1(a) Define moment of Inertia. State MI of triangular section about its base. [2]

Ans.: It is the second moment of area which is equal to product of area of the body and square of the distance of its centroid from that axis, is called as moment of Inertia.

OR

Moment of inertia of a body about an axis is defined as the sum of second moment of all elementary areas about that axis.

MI of triangular section about base $I_{base} = \frac{bh_3}{12}$

Where, b = Base of triangle and h = Height of triangle

Q.1(b) If polar moment of inertia of circular section is 2000 mm^4 then calculate diameter of the section. [2]

Ans.: Given: I_P = 2000mm^2 for circular section

$$I_P = I_{xx} + I_{yy}$$

$$I_P = \frac{\pi}{64}D^4 + \frac{\pi}{64}D^4$$

$$2000 = \frac{2\pi}{64}D^4$$

$$D = 11.946 \text{ mm}$$

Q.1(c) Define elastic body, giving two examples. [2]

Ans.: A body is said to be elastic if it regain's its original size and shape when an externally applied force causing deformation is entirely removed.

Examples

(i) Rubber band (ii) Golf Ball (iii) Soccer Ball

Q.1(d) State Hooke's law. [2]

Ans.: **Generalized Hooke's Law** : Within elastic limit, the stress and strain vary linearly.

i.e. within elastic limit : $\frac{\sigma}{\in} = \text{const.} = E$ … (1)

where **E–Young's modulus**

Q.1(e) Define slenderness ratio. **[2]**

Ans.: It is the ratio of effective length of column to minimum radius of gyration, is called as slenderness ratio.

Q.1(f) Define Resilience and modulus of resilience. **[2]**

Ans.: **Resilience:** It is the energy stored in the body or material, when loaded within elastic limit is called as strain energy or resilience.

Modulus of Resilience: It is the proof resilience per unit volume, called as modulus of resilience is called as modulus of resilience.

OR

It is the maximum strain energy stored in body per unit volume is called modulus of resilience.

Q.1(g) State meaning of effective length of column. **[2]**

Ans.:
- The material of the beam is homogeneous and isotropic i.e. the beam made of the same material throughout and it has the same elastic properties in all the directions.
- The beam is subjected to pure bending that is shear stress is totally neglected.
- The beam material is stressed within its elastic limit and thus obeys Hooke's law.
- The transverse sections which where plane before bending remains plane after bending.
- Each layer of the beam is free to expand or contact independently of the layer above or below it.
- Young's modulus (E) for the material has the same value in tension and compression.
- The radius of curvature is large as compared to the dimensions of the cross section.

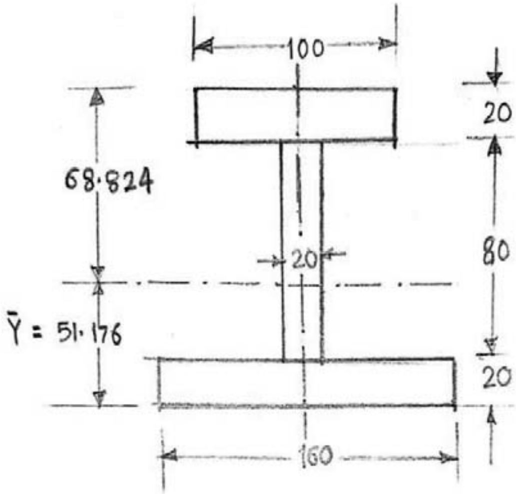
Q.2 Attempt any THREE of the following : **[12]**

Q.2(a) Determine moment of Inertia about the centroidal axes X-X and Y-Y of an Unsymmetrical I section with following details. **[4]**

Top flange - 100 mm × 20 mm
Bottom flange - 160 mm × 20 mm
Web - 80 mm × 20 mm

Ans.: **(i) Calculation of centroid:**

As given section is unsymmetrical about y-y axis,



$\overline{X}$ = $\frac{\text{Large flange width}}{2} = \frac{160}{2}$ = 80mm

A_1 = 160 × 20 = 3200mm^2, A_1 = 80 × 20 = 1600mm^2,

A_3 = 100 × 20 = 2000mm^2

Y_1 = $\frac{20}{2}$ = 10mm, Y_2 = 20 + $\frac{80}{2}$ = 60mm,

Y_3 = 20 + 30 + $\frac{20}{2}$ = 110mm,

$\overline{Y}$ = $\frac{(3200\times10)+(1600\times60)+(2000\times110)}{6800}$ = 51.17mm,

(ii) Calculation of I_{xx}:

I_{xx} = I_{xx1} + I_{xx2} + I_{xx3}

I_{xx} = $\left(I_{G1}+A_1 h_1^2\right)+\left(I_{G2}+A_2 h_2^2\right)+\left(I_{G3}+A_3 h_3^2\right)$

I_{xx} = $\left(\frac{bd^3}{12}+A_1 h_1^2\right)+\left(\frac{bd^3}{12}+A_2 h_2^2\right)+\left(\frac{bd^3}{12}+A_3 h_3^2\right)$

Here, h_1 = $\overline{Y}-Y_1$ = 1.17 – 10 = 41.17mm

h_2 = $Y_2-\overline{Y}$ = 60 – 51.17=8.83mm

h_3 = $Y_3-\overline{Y}$ = 110 – 51.17 = 58.83mm

I_{xx} = $\left(\frac{160\times20^3}{12}+3200\times41.17^2\right)+\left(\frac{20\times80^3}{12}+1600\times8.83^2\right)$

+ $\left(\frac{100\times20^3}{12}+2000\times58.83^2\right)$

I_{xx} = 13.496 × 10^{6}mm^4

(iii) Calculation of I_{yy}:

$I_{yy} = I_{yy1} + I_{yy2} + I_{yy3}$

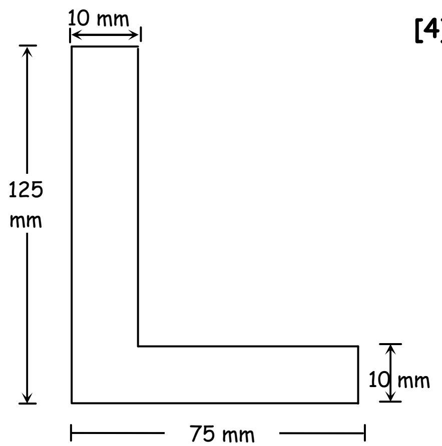
$I_{yy} = \left(I_{G1} + A_1 h_1^2\right) + \left(I_{G2} + A_2 h_2^2\right) + \left(I_{G3} + A_3 h_3^2\right)$

$I_{yy} = \left(\frac{db^3}{12}\right) + \left(\frac{db^3}{12}\right) + \left(\frac{db^3}{12}\right)$

$I_{yy} = \left(\frac{20 \times 160^3}{12}\right) + \left(\frac{80 x 20^3}{12}\right) + \left(\frac{20 \times 100^3}{12}\right)$

$I_{yy} = 8.546 \times 10^6 mm^4$

Q.2(b) Find the least moment of Inertia about the centroidal axes X–X and Y–Y of an unequal angle section 125 mm × 75 mm × 10 mm as shown in figure. **[4]**



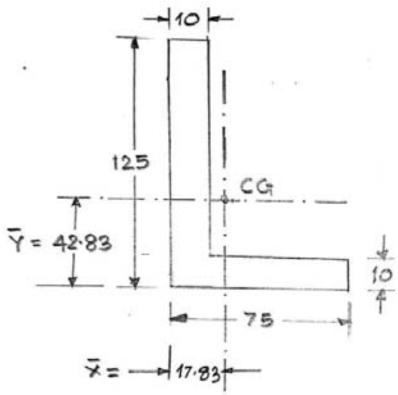
Ans.: **(i)** Calculation centroid:

$A_1 = 75 \times 10 = 750 mm^2$, $A_2 = 115 \times 10 = 1150 mm^2$

$X_1 = \frac{75}{2} = 37.5m$, $X_2 = \frac{10}{2} = 5mm$

$Y_1 = \frac{10}{2} = 5mm$, $Y_2 = 10 + \frac{115}{2} = 67.5mm$

$\overline{X} = \frac{A_1 X_1 + A_2 X_2}{A_1 + A_2} = 17.83mm$, $\overline{Y} = \frac{A_1 Y_1 + A_2 Y_2}{A_1 + A_2} = 42.83mm$



(ii) Calculation of I_{xx}:

$I_{xx} = I_{xx1} + I_{xx2}$

$I_{xx} = \left(I_{G1} + A_1 h_1^2\right) + \left(I_{G2} + A_2 h_2^2\right)$

$I_{xx} = \left(I_{G1} + A_1 h_1^2\right) + \left(I_{G2} + A_2 h_2^2\right)$

Here, $h_1 = \overline{Y} - Y = 42.28 - 5 = 37.83\text{mm}$

$h_2 = Y_2 - \overline{Y} = 67.5 - 42.83 = 24.67\text{mm}$

$$I_{xx} = \left(\frac{75\times10^3}{12} + 750\times37.83^2\right) + \left(\frac{10\times115^3}{12} + 1150\times24.67^2\right)$$

$I_{xx} = 3.046 \times 10^6\text{mm}^4$

(iii) Calculation of I_{yy}:

$I_{yy} = I_{yy1} + I_{yy2}$

$I_{yy} = \left(I_{G1} + A_1 h_3^2\right) + \left(I_{G2} + A_2 h_4^2\right)$

Here, $h_3 = \overline{X} - X = 37.5 - 17.83 = 19.67\text{mm}$

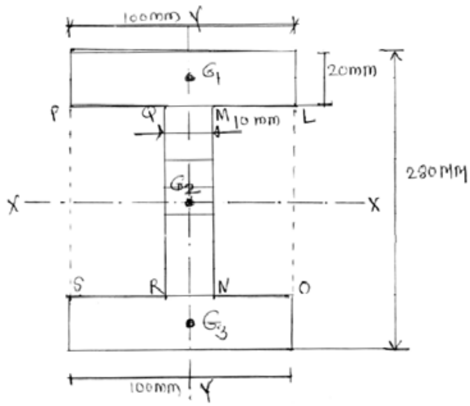
$h_4 = \overline{X} - X_2 = 17.83 - 5 = 12.83\text{mm}$

$$I_{yy} = \left(\frac{10\times75^3}{12} + 750\times19.67^2\right) + \left(\frac{115\times10^3}{12} + 1150\times12.82^2\right)$$

$I_{yy} = 840.627 \times 10^3\text{mm}^4$

Q.2(c) Find the least M.I. of a symmetrical I-section having following details: [4]
Flanges : 100 mm × 20mm
Overall depth : 280 mm
Thickness of web : 10 mm

Ans.:



I_{xx} and I_{yy} = ?

Above figure symmetrical @ XX and YY axis

So, M.I. of I – section @ xx-Axis

$M.I_{xx} = M.I._{AABCD} - M.I._{PQRS} - M.I._{LMNO}$ ... (i)

$M.I._{ABCD} = \frac{100 \times 280^3}{12} = 182.93 \times 10^6 \text{ mm}^4$

$M.I._{PQRS} = \frac{45 \times 240^3}{12} = 51.84 \times 10^6 \text{ mm}^4$

$M.I._{LMNO} = \frac{45 \times 240^3}{12} = 51.84 \times 10^6 \text{ mm}^4$

From (i)

$M.I_{xx} = 182.93 \times 10^6 - [2 \times 51.84 \times 16^6]$

$M.I_{xx} = 79.25 \times 10^6 \text{ mm}^4$

M.I. of I- section @ Y – Y axis

$I_{yy} = I_{yy1} + Iyy2 + I_{yy3}$ ... (ii)

$I_{yy1} = \frac{\partial b^3}{12} = \frac{20 \times 100^3}{12} = 1.67 \times 10^6 \text{mm}^4$

$I_{yy2} = \frac{\partial b^3}{12} = \frac{240 \times 10^3}{12} = 20 \times 10^3 \text{mm}^4$

$I_{yy3} = I_{yy1} = 1.67 \times 10^6 \text{ mm}^4$

From equation (ii)

$I_{yy} = 1.67 \times 10^6 + 20 \times 10^3 + 1.67 \times 10^6$

$I_{yy} = 3.36 \times 10^6 \text{ mm}^4$ least M.I.

Q.2(d) A column having diameter 200 mm is of length 3 meters. Both ends of a column are hinged. Find Euler's crippling load. Take $E = 2 \times 10^5$ MPa. **[4]**

Ans.: Given: d = 200 mm

L = 3m = 3000mm

$E = 2 \times 10^5$ mPa

P_E = Euler's crippling load = ?

∵ Both end of column are hinged

Le = L 3000 mm

We have

Euler's crippling load

$P_E = \frac{\pi^2 EI}{(Le)^2}$ ... (i)

For circular column, M.I. is

$I = \frac{\pi}{64}\partial 4$

$$I = \frac{\pi}{64} \times (200)^4$$

$I = 78.53 \times 10^6\ mm^4$

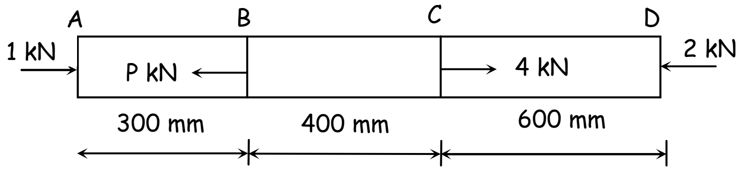
From equation (i)

$$P_E = \frac{\pi^2 \times 2 \times 10^5 \times 78.53 \times 10^6}{(3000)^2}$$

$P_E = 17.22 \times 10^6 N$

Q.3 Attempt any THREE of the following : [12]

Q.3(a) A bar of uniform cross sectional area 100 mm^2 is subjected to axial forces as shown in Fig. Calculate the net change in length of the bar. Take E = 2 × 10^5 N/mm^2. [4]



Ans.: $A = 100\ mm^2$ $E = 2 \times 10^5 N/mm^2$ $\delta L = ?$

To calculate, P

1 + 4 = P – 2

P = 5 – 2

P = 3kN

$\delta L = -\ \delta L_{AB} + \delta L_{BC} - \delta L_{CD}$

$$\delta L = -\left(\frac{PL}{AE}\right)_{AB} + \left(\frac{PL}{AE}\right)_{BC} - \left(\frac{PL}{AE}\right)_{CD}$$

$$\delta L = -\left(\frac{1 \times 10^3 \times 300}{100 \times 200 \times 10^3}\right)_{AB} + \left(\frac{2 \times 10^3 \times 400}{100 \times 200 \times 10^3}\right)_{BC} - \left(\frac{2 \times 10^3 \times 600}{100 \times 200 \times 10^3}\right)_{CD}$$

$\delta L = 0.015 + 0.04 – 0.06$

$\delta L = -\ 0.035mm$

Q.3(b) A steel tube with 40 mm inside diameter and 4 mm thickness is filled with concrete. Determine load shared by each material due to axial thrust of 60 kN. [4]

Take E_{steel} = 210 × 10^3 N/mm^2

$E_{concrete}$ = 14 × 10^3 N/mm^2

Ans.: **Given:**

d = 400mm t = 4mm P = 60KN

$E_S = 210 \times 10^3\ N/mm^2$ $E_C = 14 \times 10^3\ N/mm^2$

D = d + 2t = 40 + (2 × 4)

D = 48 mm

$A_s = \frac{\pi}{4}\times(D^2 - d^2)$

$A_s = \frac{\pi}{4}\times(48^2 - 40^2)$

$A_s = 552.92 mm^2$

$A_C = \frac{\pi}{4}\times(d^2)$

$A_c = \frac{\pi}{4}\times(40^2)$

$Ac = 1256.637 mm^2$

$m = \frac{E_S}{E_C} = \frac{210\times10^3}{14\times10^3} = 15$

$es = ec$

$\frac{\sigma_s}{E_S} = \frac{\sigma_c}{E_C}$ ... (i)

$\sigma_s = \left(\frac{E_S}{E_C}\right)\sigma_c$

$\sigma_s = 15\sigma_c$

$P = P_S + P_C$ ... (ii)

$P = \sigma_s A_S + \sigma_c A_C$

$60 \times 10^3 = 15\sigma_c \times 552.92 + \sigma_c \times 1256.637$

$\sigma_c = \frac{60\times10^3}{9550.437}$

$\sigma_c = 6.2824\ N/mm^2$

$P_c = \sigma_c A_C$

$P_c = 6.2824 \times 1256.637$

$P_c = 7894.74 N$

$P_c = 7.89 kN$

$\sigma_s = 15\sigma c$

$\sigma_s = 15 \times 6.2824$

$\sigma_s = 94.236\ N/mm^2$

$P_s = \sigma_s A_S$

$P_s = 94.236 \times 552.92$

$P_s = 52104.96 N$

$P_s = 52.104 kN$

Q.3(c) Draw stress-strain curve for mild steel under tensile loading showing important points on it. **[4]**

Ans.:

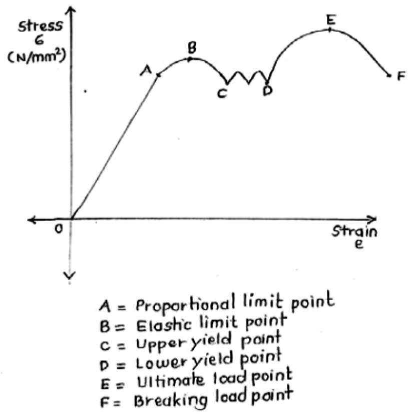


Fig. Strain curve for Mild Steel

Q.3(d) State any four assumptions made in theory of pure bending. **[4]**

Ans.: Assumption in the theory of pure bending :

(i) The material of the beam is homogenous & isotrophic.
(ii) The beam material is stressed to obey Hooke's law.
(iii) The transverse sections, which were plane before bending, remain plane after bending.
(iv) Each layer of beam is free to expand or contract independently.
(v) Value of E is the same in tension & compression.

Q.4 Attempt any TWO of the following : **[16]**

Q.4(a) A cube of 200 mm side is subjected to a compressive force of 3600 kN on all its faces. The change in the volume of cube is found to be 5000 mm^3. Calculate the Bulk modulus. If μ = 0.28, find the Young's modulus. **[8]**

Ans.: $\sigma = \frac{P}{A} = \frac{300 \times 10^3}{200 \times 200}$

$\sigma = 90 N/mm^2$

$V = L^3 = (200)^3$

$V = 8 \times 10^6 \; mm^3$

$$\frac{\delta V}{V} = \frac{\sigma x + \sigma y + \sigma z}{E}(1 - 2\mu)$$

$$E = \frac{3\sigma}{\left(\frac{\delta V}{V}\right)}(1 - 2\mu)$$

$$E = \frac{3 \times 90}{\left(\frac{5000}{8 \times 10^6}\right)(1 - 2 \times 0.28)}$$

$E = 1.9 \times 10^5 N/mm^2$

Q.4(b) An over changing beam ABC is such that AB = 4m, BC = 1 m, is supported at A and B. The beam ABC is subjected to vdl of 30 kN/m over the entire length ABC. It is subjected to point load 50 kN at the free end C. Draw SFD and BMD with calculations and locate the point of contra flexure. **[8]**

Ans.: (i) To calculate the reactions at supports:

$R_B \times 4 = (30 \times 5 \times 2.5) + 50 \times 5$

$R_B = 156325kN$

$R_A = (30 \times 5 \times 50) - 156.25$

$R_A = 43.75kN$

(ii) Shear force calculations

SF at A = 43.75 kN

SF at B_L = 43.75 – 30 × 40 = -76.25 kN

SF at B_R = - 76.25 + 156.25 = 80 kN

SF at C_L = 80 – 30 × 1 = 50 kN

SF at C = 50 – 50 = 0 $(\therefore OK)$

(iii) Bending moment calculations

BM at A and C = 0

BM at B = $-50 \times 1 - 30 \times 1 \times \frac{1}{2}$

= - 65 kN-m

(i) To calculate Maximum Bending Moment

SF at x = 0

$\therefore$ 43.75 – 30 × = 0

$\therefore$ x = 1.458m from A

$BM_{max} = 43.75 \times 1.458 - 30 \times \frac{1.458^2}{2}$

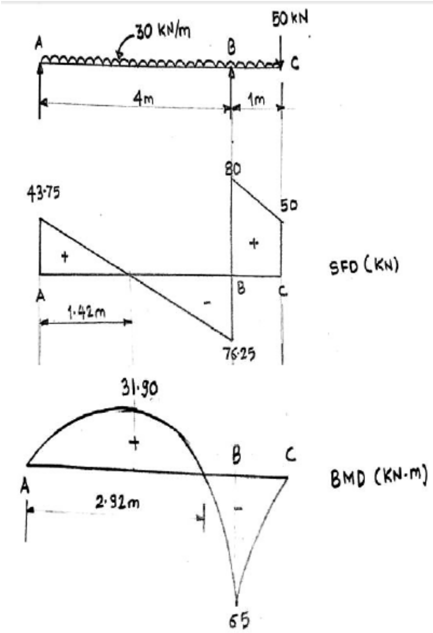
BM_{max} = 31.90kN – m

(ii) To locate point of contraflexure

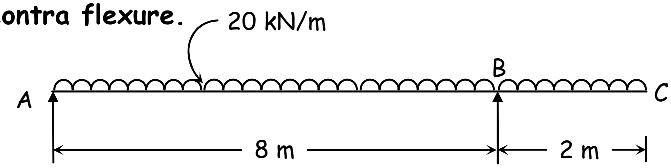
BM at x′ = 0

$43.75\,x' - 30 \times \frac{x'}{2} = 0$

X′ = 2.916 m from A



Q.4(c) Draw SFD and BMD of a beam as shown in figure. Also find the point of contra flexure. [8]



Ans.: (i) To find reactions R_A and R_B

$R_A + R_B = 20 \times 10$

$R_A + R_B = 200$ … (i)

$\Sigma M_A = 20 \times 10 \times \frac{10}{2} - R_B \times 8$

$\Sigma M_A = 1000 - 8 R_B$

$8R_B = 1000$

$R_B = 125$ kN

$\therefore$ $R_A = 200 - R_B$

$R_A = 200 - 125$

$R_A = 75$ kN

(ii) Shear force calculation

$F_C = 0$

$F_{BR} = 20 \times 2 = 40$ kN

$F_{BL} = 40 - 125$

$F_{BL} = -85$ kN

F_A = – 85 + 20 × 8
F_A = – 85 + 160
F_A = 75 kN

(iii) B.M. calculation

M_C = 0 kN.m

$M_B = -(20 \times 2) \times \frac{2}{2}$

$= -20 \times 2 \times 1$

$= -40$ kN.m

M_A = 0 kN.m

(iv) To locate the position of point of contrashear

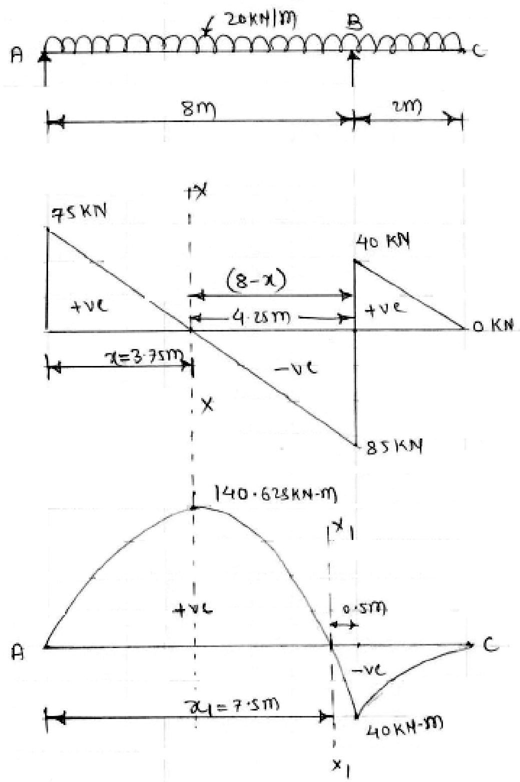
$\frac{85}{8-x} = \frac{75}{x}$

85x = 75 (8 – x)

85x = 600 – 75x

∴ 160x = 600

x = 3.75 m from left



$M_{XX} = M_{PCS} = 75 \times x - 20 \times x \times \frac{x}{2}$

$M_{XX} = 75 \times 3.75 - 20 \times 3.75 \times \frac{3.75}{2}$

M_{XX} = 281.25 – 140.625

M_{XX} = 140.625 kN.m (sagging)

To locate the point of contraflexure (P_{CF})

$M_{x_1} = 75\,x_1 - 20 \times x_1 \times \frac{x_1}{2}$

$0 = 75\,x_1 - 10x_1^2$

$0 = x_1 (75 - 10x_1)$

∴ $75 = 10x_1 = 0$

$75 = 10x_1$

x_1 = 7.5 m from A

Point of contraflexrure (P_{CF}) = 7.5 cm from 'A'

Q.4(d) State the flexural formula, giving meaning of the symbols used in it. [4]

Ans.: Flexural formula

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Where

m = max bending moment which is equal to moment of resistance of beam.

I = M.I. of beam section about the neutral axis since neutral axis always lies at the centroid of the section.

$I = I_{NA} = I_{XX}$

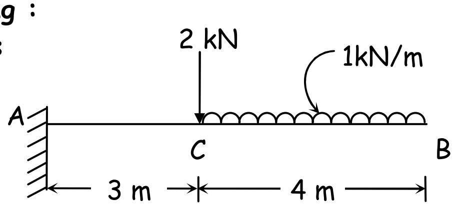
σ = Bonding stress in a layer at a distance y from N.A.

Y = distance of the layer from the N.A. of the beam material.

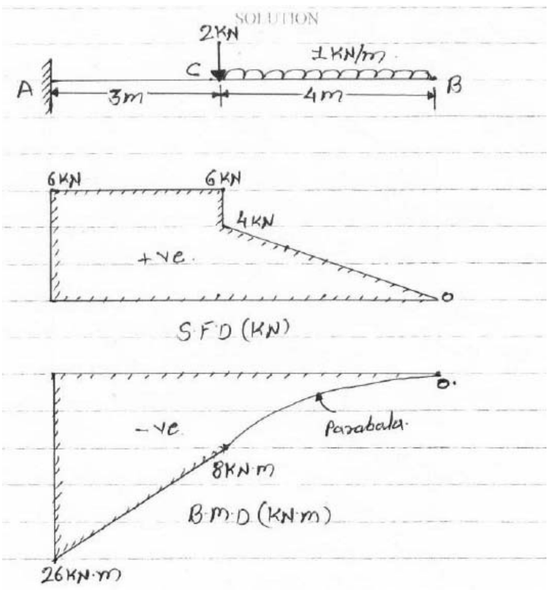
R = Radius of curvature of the bentup beam.

Q.5 Attempt any TWO of the following : [12]

Q.5(a) A cantilever beam is loaded as shown in figure. Draw the S.F.D. and B.M.D. [6]



Ans.:



(i) Support reaution

$\sum Fy = 0$

RA = 2 + (1 × 4) = 6kN

(ii) Shear force calculation [↑+ve ↓-ve]

S.F at just left of A = 0 kN

S.F at just right of A = R_A = 6 kN
S.F at just left of C = 6kN
S.F. at just right of C = 6 – 2 = 4 kN
S.F at B = 4 – (1 × 4) = 0 kN

Q.5(b) A T section of flange 160 mm x 20 mm and web 180 x 20 mm is simply supported at the both ends. It carries two concentrated loads of 100 kN each acting 2m distance from each support. Span of the beam is 8m. Determine the maximum bending stress induced in the beam and draw bending stress distribution diagram and also find bending stress at the layer 100 mm from the bottom. [6]

Ans.: $\bar{y} = \dfrac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \dfrac{(160 \times 20) \times 190 + (180 \times 20) \times 90}{3200 + 3600}$

$\bar{y}$ = 137.05 mm from base

$I_{NA} = I_{xx} = I_{xx1} + I_{xx2}$

$I_{NA} = I_{xx} = \left(I_{G1} + A_1 h_1^2\right) + \left(I_{G2} + A_2 h_2^2\right)$

$I_{NA} = I_{xx} = \left(\dfrac{bd^3}{12} + A_1 h_1^2\right) + \left(\dfrac{bd^3}{12} + A_2 h_2^2\right)$

$I_{NA} = I_{xx} = \left(\dfrac{160 \times 20^3}{12} + 3200 \times 52.94^2\right) + \left(\dfrac{20 \times 180^3}{12} + 3600 \times 47.06^2\right)$

$I_{NA} = I_{xx} = 26767843.15 \text{ mm}^4$

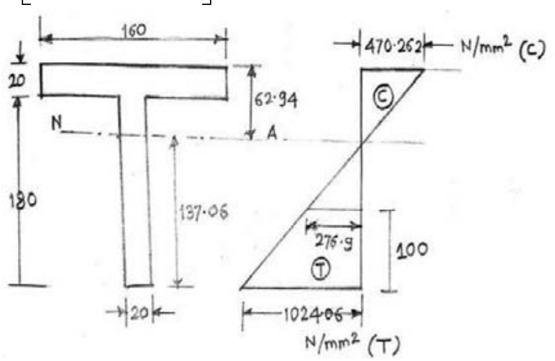
Maximum bending moment

M = 100 × 2 = 200 kN-m = 200 × 10^6N – mm

$\sigma_c = \dfrac{M}{I} y_c = \left[\dfrac{200 \times 10^6}{26767843.15}\right] \times 62.94 = 470.265 \text{ N/mm}^2 (C)$

$\sigma_t = \dfrac{M}{I} y_t = \left[\dfrac{200 \times 10^6}{26767843.15}\right] \times 137.06 = 1024.06 \text{ N/mm}^2 \text{ (T)}$

$\sigma_t = \dfrac{M}{I} y_{t(100)} = \left[\dfrac{200 \times 10^6}{26767843.15}\right] \times 37.06 = 276.899 \text{ N/mm}^2 (T)$



Q.5(c) A metal rod 20 mm diameter and 2 m long when subjected to tensile force of 60 kN shows an elongation of 2 mm and reduction in diameter 0.006 mm. Calculate the modulus of elasticity and modulus of rigidity. [6]

Ans.: $E = \frac{PL}{A\delta L}$

$E = \frac{60\times10^3\times2\times10^5}{\frac{\pi}{4}\times20^2\times2}$

$E = 1.91 \times 105\ N/mm^2$

$\mu = \frac{\text{Lateral Strain}}{\text{Linear Strain}}$

$\mu = \frac{\left(\frac{\delta d}{d}\right)}{\left(\frac{\delta L}{L}\right)} = \frac{\left(\frac{0.006}{20}\right)}{\left(\frac{2}{2000}\right)}$

$\mu = 0.3$

$E = 2G(1 + \mu)$

$= 2G(1 + 0.3)$

$1.91 \times 10^5 = 2G(1 + 0.3)$

$G = \frac{1.91\times10^5}{2\times1.3}$

$G = 7.345 \times 10^4 N/mm^2$

Q.6 Attempt any TWO of the following : [12]

Q.6(a) (i) A simply supported beam of span `L' carries central point load `W'. Draw SED and BMD [6]

(ii) Define shear force and bending moment. Write unit of each. Also state relation between them.

Ans.: (i) Step 1

Calculation of Reaction,

As, the load is at centre so,

Support reaction are equal'

$R_A = R_B = \frac{W}{2}$

Step 2

Share force calculation

(a) S.F. at any section between A and C is,

$F_x = + R_A = \frac{W}{2}$

(b) S.F. at any section between B and C is,

$$F_x = -R_B = -\frac{W}{2}$$

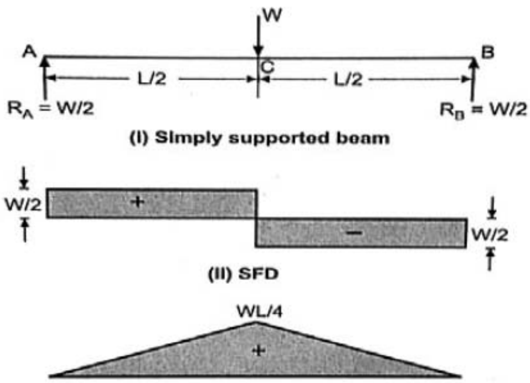
Step 3 : Bending Moment Calculation

Beam is simply supported at the end A and B,

$\therefore \quad M_A = M_B = 0$

$\therefore \quad M_{max} = MC = +\frac{W}{2}\times\frac{L}{2}$

$\therefore \quad M_C = \frac{WL}{4}$



(ii) **Shear force:** Shear force at any cross section of the beam is the algebraic sum of vertical forces on the beam acting on right side or left side of the section is called as shear force.

OR

A shear force is the resultant vertical force acting on the either side of a section of a beam.

Unit :- kN or N

Bending Moment: Bending moment at any section at any cross section is the algebraic sum of the moment of all forces acting on the right or left side of section is called as bending moment.

Unit: kN-m or N-m

Relation between shear force and bending movement

$$\frac{dM}{dx} = F$$

The rate of change of bending moment at any section is equal to the shear force at that section.

Q.6(b) A cast iron column 100 mm external diameter and 80 mm internal diameter is 2 m long. It is fixed at one end and hinged at other end. Calculate the safe axial load by Rankine's formula taking factor of safety 3. Assume σ_c = 550 N/mm^2 and Rankine's constant $\alpha = \frac{1}{1600}$. **[6]**

Ans.: ***Given***

D = 100mm, d = 80mm, L = 2m = 2000mm,

FOS = 3, σ_c = 550N/mm^2, $\alpha = \frac{1}{1600}$

As, the column is fixed at one end and hinged at other end.

Efeective length, (Le) = $\frac{L}{\sqrt{2}} = \frac{2000}{\sqrt{2}}$

$$Le = 1414.2mm$$

For hollow circular column.

$I_{min} = I_{xx} = I_{yy} = \frac{\pi}{64}\left(100^4 - 80^4\right)$

$I_{min} = I_{xx} = I_{yy} = 2898119.223mm^4$

Area,

$A = \frac{\pi}{4}\left(100^2 - 80^2\right)$

$A = 2827.433mm^2$

$K^2 = \frac{I}{A}$

$K^2 = \frac{2898119.223}{2827.433}$

K = 32.056mm

$\therefore \quad 1 + a\frac{(Le)^2}{K^2} = 1 + \left(\frac{1}{1600}\right)\times\left(\frac{(1414.2)^2}{32.0156}\right)$

$1 + a\frac{(Le)^2}{K^2} = 2.2195$

By using Rankine's formula,

$P_R = \frac{\sigma_c A}{1 + a\frac{(Le)^2}{K^2}}$

$P_R = \frac{550\times 2827.433}{2.2195}$

$P_R = 707644.2077N$

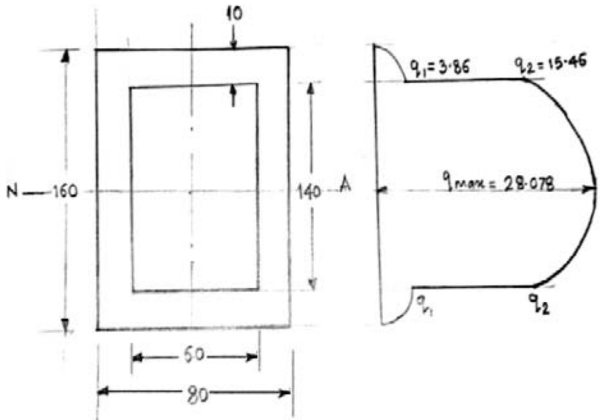
$$\text{Safe Load} = \frac{\text{Rankine's crippling load}}{\text{Factor of safety}}$$

$$\text{Safe Load} = \frac{707644.2077}{3}$$

Safe Load = 233548.0692N
Safe Load = 233.548kN

Q.6(c) A beam has hollow rectangular section with external dimensions 80 mm × 160 mm and uniform thickness of section is 10 mm. Draw shear stress variation diagram. It section is subjected to the shear force 70 kN. Also determine ratio of maximum shear stress and average shear stress. **[6]**

Ans.:



$A = (BD - b\,d) \quad = (80 \times 160 - 60 \times 140)$
$A = 4400\ mm^2$

$$I_{NA} = \frac{1}{12}\left(BD^3 - bd^3\right) = \frac{1}{12}(80 \times 160^3 - 60 \times 140^3)$$
$I_{NA} = 13586666.67\ mm^4$

$$q_{avg} = \frac{S}{A} = \frac{70 \times 10^3}{4400}$$
$q_{avg} = 15.91\ N/mm^2$

$$q_1 = \frac{SA\bar{Y}}{bI} = \frac{70 \times 10^3 \times (80 \times 10) \times 75}{80 \times 13586666.67}$$
$q_1 = 3.864\ N/mm^2$

$q_2 = q_1 \times \frac{80}{20} = 3.864 \times 40$

$q_2 = 15.456 \text{ N/mm}^2$

$q_{add} = \frac{SA\overline{Y}}{bI} = \frac{70 \times 10^3 \times 2(70 \times 10) \times 35}{20 \times 13586666.67}$

$q_{add} = 12.622 \text{ N/mm}^2$

$q_{NA} = q_{max} = q_2 + q_{add}$

$= 15.456 + 12.622$

$q_{NA} = 28.078 \text{ N/mm}^2$

Ratio,

$\frac{q_{max}}{q_{avg}} = \frac{28.078}{15.91}$

$\frac{q_{max}}{q_{avg}} = 1.765$

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