

**Engineering Mathematics**

Time: 3 Hrs.]

Prelim Question Paper

[Marks : 100

- Instructions :**
- (1) All Questions are Compulsory.
  - (2) Answer each next main Question on a new page.
  - (3) Illustrate your answers with neat sketches wherever necessary..
  - (4) Figures to the right indicate full marks.
  - (5) Assume suitable data, if necessary.
  - (6) Mobile Phone and any other Electronic Communication devices are not permissible in Examination Hall.

1. Attempt any **TEN** of the following :

[20]

- (a) If  $(3x - 4y) + i(x + y) = 7$ , find  $x, y$ .
- (b) Express in the form  $a + ib$ .  $\frac{1+i}{2-i}$  where  $a, b \in \mathbb{R}, i = \sqrt{-1}$ .
- (c) If  $f(x) = 3x^2 - 5x + 7$  show that  $f(-1) = 3f(1)$ .
- (d) State whether the function  $f(x) = \frac{e^x + e^{-x}}{2}$  is even or odd.
- (e) Evaluate  $\lim_{x \rightarrow 0} \frac{2^x - 1}{\sin 2x}$ .
- (f) Evaluate  $\lim_{x \rightarrow 0} \frac{3^x - 4^x}{x}$ .
- (g) If  $y = e^x \cdot \sin x \cdot \cos x$  find  $\frac{dy}{dx}$ .
- (h) Find  $\frac{dy}{dx}$  if  $y = \log(x^2 + 2x)$ .
- (i) Find  $\frac{dy}{dx}$  if  $x = \sin \theta, y = \cos \theta$ .
- (j) If  $x^2 + y^2 = 4$  find  $\frac{dy}{dx}$ .
- (k) Show that root of equation  $x^3 - 2x - 5 = 0$  lies between 2 and 3.
- (l) Find first iteration by Jacobi's method :  
 $10x + y + 2z = 13, \quad 3x + 10y + z = 14, \quad 2x + 3y + 10z = 15$

2. Attempt any **FOUR** of the following :

[16]

- (a) If  $f(x) = \frac{x-4}{4x-1}$  then show that  $f[f(x)] = x$ .
- (b) Express the following number in polar form  $\frac{-1}{2} + \frac{\sqrt{3}}{2}i$ .
- (c) Find all cube root of unity.
- (d) Simplify using De-Moivre's theorem :  
$$\frac{(\cos 5\theta - i \sin 5\theta)^{\frac{2}{5}} \left( \cos \frac{2}{7}\theta + i \sin \frac{2}{7}\theta \right)^7}{(\cos 4\theta + i \sin 4\theta)^{\frac{1}{4}} \left( \cos \frac{2}{3}\theta - i \sin \frac{2}{3}\theta \right)^3}$$
- (e) If  $f(x) = x^2 - 4x + 11$ , solve the equation  $f(x) = f(3x - 1)$ .
- (f) Simplify  $i + i^{10} + i^{50} + i^{100}$ .

3. Attempt any **FOUR** of the following : [16]
- (a) If  $f(x) = ax^2 + bx + 3$  and  $f(1) = 4$ ,  $f(2) = 11$ , find 'a' and 'b'.
- (b) If  $f(x) = \log \left( \frac{1+x}{1-x} \right)$  then prove that  $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$ .
- (c) Evaluate  $\lim_{x \rightarrow 1} \frac{x^3 + 3x^2 - 6x + 2}{x^3 + 3x^2 - 3x - 1}$ .
- (d) Evaluate  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin^2 x - \cos^2 x}{1 - \tan x}$ .
- (e) Evaluate  $\lim_{x \rightarrow \infty} \left( \sqrt{x^2 + 5x} - x \right)$ .
- (f) Evaluate  $\lim_{x \rightarrow 0} \frac{6^x - 3^x - 2^x + 1}{x^2}$ .

4. Attempt any **FOUR** of the following : [16]
- (a) If  $y = \sin^{-1}(3x - 4x^3)$  find  $\frac{dy}{dx}$ .
- (b) Using first principle find derivative of  $f(x) = a^x$ .
- (c) If  $u$  and  $v$  are differentiable functions of  $x$  and  $y = u \cdot v$  then prove that  

$$\frac{dy}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$
- (d) Differentiate w.r.t  $x$ ,  $\tan^{-1} \left( \frac{5x}{1-6x^2} \right)$ .
- (e) Find  $\frac{dy}{dx}$  if  $13x^2 + 2x^2y + y^3 = 1$ .
- (f) If  $y = \tan^{-1} \left( \frac{2t}{1-t^2} \right)$  and  $x = \left( \sin^{-1} \left( \frac{2t}{1+t^2} \right) \right)$  find  $\frac{dy}{dx}$ .

5. Attempt any **FOUR** of the following : [16]
- (a) Evaluate  $\lim_{x \rightarrow 1} \frac{\sin \pi x}{x-1}$ .
- (b) Evaluate  $\lim_{x \rightarrow 3} \frac{\log x - \log 3}{x-3}$ .
- (c) Using Bisection method find the approximate root of  $x^2 + x - 3 = 0$  (3 iterations).
- (d) Using False Position method find the root of  $x^3 - x - 4 = 0$  (3 iterations only).
- (e) Using Newton-Raphson method find the root of  $x^4 - x - 9 = 0$  (carry out 3 iterations).
- (f) Using Newton-Raphson method find approximate value of  $\sqrt{10}$  (3 iterations).

6. Attempt any **FOUR** of the following : [16]
- (a) If  $y = \sin 5x - 3 \cos 5x$  show that  $\frac{d^2y}{dx^2} + 25y = 0$ .
- (b) If  $x = a(\theta - \sin \theta)$ ,  $y = a(1 - \cos \theta)$  find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{4}$ .
- (c) Solve by Jacobi's method performing 3 iterations :  
 $20x + y - 2z = 17$ ,  $3x + 20y - z = -18$ ,  $2x - 3y + 20z = 25$
- (d) Solve by Gauss-Seidal method (3 iterations)  
 $15x + 2y + z = 18$ ,  $2x + 20y - 3z = 19$ ,  $3x - 6y + 25z = 22$
- (e) Solve by Gauss Elimination method :  
 $x + 2y + 3z = 14$ ,  $3x + y + 2z = 11$ ,  $2x + 3y + z = 11$
- (f) Solve by Gauss-Seidal method (2 iterations)  
 $5x - y = 9$ ,  $x - 5y + z = -4$ ,  $y - 5z = 6$ , Taking  $x_0 = 1.5$ ,  $y_0 = 0.5$ ,  $z_0 = -0.5$

**Paper Discussion Schedule for all Subject: F.Y. Diploma Sem.-II**

<b>Date</b>	<b>Day</b>	<b>Timing</b>	<b>Centre</b>
9 April 2017	Sunday	9 a.m. to 11 a.m.	Dadar
9 April 2017	Sunday	12 p.m.to 2 p.m.	Thane
9 April 2017	Sunday	9 a.m. to 11 a.m.	Ghatkopar
9 April 2017	Sunday	12 p.m. to 2 p.m.	Borivali
9 April 2017	Sunday	12 p.m. to 2 p.m.	Nerul
9 April 2017	Sunday	3 pm to 5 pm	Kalyan

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