Engineering Mathematics

Time: 3 Hrs.] Prelim Question Paper Solution **[Marks : 100**

Q.1 Attempt any TEN of the following : **[20]**

Q.1(a) If (3x − 4y) + i (x + y) = 7, find x, y. **[2]**

(A) $(3x - 4y) + i(x + y) = 7$

$\therefore (3x - 4y) + i(x + y) = 7 + 0i$

$\therefore 3x - 4y = 7$ and $x + y = 0$

$\therefore 3x - 4y = 7$

$\underline{4x + 4y = 0}$

$\therefore 7x = 7$

$\therefore x = 1$

$\therefore y = -1$

Q.1(b) Express in the form a + ib. $\frac{1+i}{2-i}$ where a, b $\in$ R, i = $\sqrt{-1}$. **[2]**

(A) $\frac{1+i}{2-i} = \frac{1+i}{2-i}\times\frac{2+i}{2+i}$

$= \frac{2+i+2i+i^2}{2^2-i^2}$

$= \frac{2+3i-1}{4-(-1)}$

$= \frac{1+3i}{5}$ or $\frac{1}{5}+\frac{3}{5}i$

Q.1(c) If f(x) = 3x^2 − 5x + 7 show that f(−1) = 3f(1). **[2]**

(A) $\therefore f(-1) = 3(-1)^2 - 5(-1) + 7$

$= 15$

$f(1) = 3(1)^2 - 5(1) + 7$

$= 5$

$\therefore f(-1) = 3f(1)$

Q.1(d) State whether the function f(x) = $\frac{e^x + e^{-x}}{2}$ is even or odd. **[2]**

(A) $f(-x) = \frac{e^{-(-x)} + e^{-x}}{2}$

$= \frac{e^x + e^{-x}}{2}$

$= f(x)$

$\therefore$ f(x) is even.

Q.1(e) Evaluate $\lim\limits_{x\to 0}\frac{2^x-1}{\sin 2x}$. **[2]**

(A) $\lim\limits_{x\to 0}\frac{2^x-1}{\sin 2x} = \lim\limits_{x\to 0}\frac{2^x-1}{x}\times\frac{x}{\sin 2x}$

$= \lim\limits_{x\to 0}\frac{2^x-1}{x}\times\frac{2x}{\sin 2x}\times\frac{1}{2}$

$$= \log 2 \times 1 \times \frac{1}{2}$$

$$= \frac{1}{2} \log 2$$

Q.1 (f) Evaluate $\lim_{x \to 0} \frac{3^x - 4^x}{x}$. **[2]**

(A) $\lim_{x \to 0} \frac{3^x - 4^x}{x} = \lim_{x \to 0} \frac{3^x - 1 - 4^x + 1}{x}$

$$= \lim_{x \to 0} \frac{(3^x - 1) - (4^x - 1)}{x}$$

$$= \lim_{x \to 0} \left[\frac{3^x - 1}{x} - \frac{4^x - 1}{x} \right]$$

$$= \log 3 - \log 4$$

Q.1(g) If y = $e^x \cdot \sin x \cdot \cos x$ find $\frac{dy}{dx}$. **[2]**

(A) $\therefore \frac{dy}{dx} = e^x \cdot \sin x \cdot \frac{d}{dx}(\cos x) + e^x \cdot \cos x \frac{d}{dx}(\sin x) + \sin x \cdot \cos x \frac{d}{dx}(e^x)$

$$= e^x \cdot \sin x \cdot (-\sin x) + e^x \cdot \cos x \cdot \cos x + \sin x \cdot \cos x \cdot e^x$$

$$= -e^x \cdot \sin^2 x + e^x \cdot \cos^2 x + e^x \cdot \sin x \cdot \cos x$$

$$= e^x \cdot (-\sin^2 x + \cos^2 x + \sin x \cdot \cos x)$$

OR

$$y = e^x \cdot \sin x \cdot \cos x = \frac{1}{2} \cdot e^x \cdot \sin 2x$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\left[e^x \cdot \frac{d}{dx}(\sin 2x) + \sin 2x \cdot \frac{d}{dx}(e^x) \right]$$

$$= \frac{1}{2}\left[e^x \cdot \cos 2x \cdot 2 + \sin 2x \cdot e^x \right]$$

$$= \frac{1}{2} e^x \cdot (2\cos 2x + \sin 2x)$$

Q.1(h) Find $\frac{dy}{dx}$ if y = log (x^2 + 2x). **[2]**

(A) y = log(x^2 + 2x)

$$\therefore \frac{dy}{dx} = \frac{1}{x^2 + 2x} \cdot \frac{d}{dx}(x^2 + 2x)$$

$$= \frac{1}{x^2 + 2x} \times (2x + 2)$$

Q.1 (i) Find $\frac{dy}{dx}$ if x = sin θ, y = cos θ. **[2]**

(A) x = sin θ, y = cosθ

$$\therefore \frac{dx}{d\theta} = \cos\theta \quad \text{and} \quad \frac{dy}{d\theta} = -\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$= \frac{-\sin\theta}{\cos\theta}$$

$$= -\tan\theta$$

Q.1(j) If $x^2 + y^2 = 4$ find $\frac{dy}{dx}$. **[2]**

(A) $x^2 + y^2 = 4$

$$\therefore\ 2x + 2y\frac{dy}{dx} = 0$$

$$\therefore\ 2y\frac{dy}{dx} = -2x$$

$$\therefore\ \frac{dy}{dx} = -\frac{x}{y}$$

Q.1(k) Show that root of equation $x^3 - 2x - 5 = 0$ lies between 2 and 3. **[2]**

(A) $f(x) = x^3 - 2x - 5$

$\therefore\ f(2) = -1 < 0$

$f(3) = 16 > 0$

Therefore the root lies between 2 and 3

Q.1(l) Find first iteration by Jacobi's method : **[2]**

$10x + y + 2z = 13$, $3x + 10y + z = 14$, $2x + 3y + 10z = 15$

(A) $10x + y + 2z = 13$,

$3x + 10y + z = 14$,

$2x + 3y + 10z = 15$

$$\therefore\ x = \frac{13 - y - 2z}{10}$$

$$y = \frac{14 - 3x - z}{10}$$

$$z = \frac{15 - 2x - 3y}{10}$$

Now we start with : $x_0 = 0 = y_0 = z_0$.

$$\therefore\ x_1 = \frac{13 - (0) - 2(0)}{10} = 1.3$$

$$y_1 = \frac{14 - 3(0) - (0)}{10} = 1.4$$

$$z_1 = \frac{15 - 2(0) - 3(0)}{10} = 1.5$$

Q.2 Attempt any FOUR of the following : **[16]**

Q.2(a) If $f(x) = \frac{x-4}{4x-1}$ then show that $f[f(x)] = x$. **[4]**

(A) $f(x) = \frac{x-4}{4x-1}$

$$\therefore\ f[f(x)] = \frac{f(x) - 4}{4f(x) - 1}$$

$$= \frac{\frac{x-4}{4x-1} - 4}{4\left(\frac{x-4}{4x-1}\right) - 1}$$

$$= \frac{\dfrac{x-4-4(4x-1)}{4x-1}}{\dfrac{4(x-4)-1(4x-1)}{4x-1}}$$

$$= \frac{x-4-16x+4}{4x-16-4x+1}$$

$$= \frac{-15x}{-15}$$

$$= x$$

Q.2(b) Express the following number in polar form $\frac{-1}{2}+\frac{\sqrt{3}}{2}i$. **[4]**

(A) $\therefore\ r = \sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = 1$

$$\theta = 180^\circ - \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) = 180^\circ - 60^\circ = 120^\circ \text{ or}$$

or $\theta = \pi - \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\therefore\ z = r(\cos\theta + i\sin\theta)$

$= \cos 120^\circ + i\sin 120^\circ$ or $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$

Q.2(c) Find all cube root of unity. **[4]**

(A) Let $z = 1 = 1 + 0i$ $\quad a = 1,\ b = 0$

$\therefore\ r = \sqrt{1^2+0^2} = 1$

$\theta = \tan^{-1}\left(\frac{0}{1}\right) = 0$

$\therefore\ z = 1 = r(\cos\theta + i\sin\theta)$

$= \cos 0 + i\sin 0$

$= \cos 2\pi k + i\sin 2\pi k$

$\therefore\ 1^{1/3} = [\cos 2\pi k + i\sin 2\pi k]^{1/3}$

$= \cos\left(\frac{2\pi k}{3}\right) + i\sin\left(\frac{2\pi k}{3}\right)$

For $k = 0$, $\quad 1^{1/3} = \cos(0) + i\sin(0)$

$= 1 + 0 = 1$

For $k = 1$, $\quad 1^{1/3} = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$

$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

For $k = 2$, $\quad 1^{1/3} = \cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right)$

$= -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

Q.2(d) Simplify using De-Moivre's theorem : **[4]**

$$\frac{(\cos 5\theta - i\sin 5\theta)^{\frac{2}{5}}\left(\cos\frac{2}{7}\theta + i\sin\frac{2}{7}\theta\right)^{7}}{(\cos 4\theta + i\sin 4\theta)^{\frac{1}{4}}\left(\cos\frac{2}{3}\theta - i\sin\frac{2}{3}\theta\right)^{3}}$$

(A)
$$\frac{(\cos 5\theta - i\sin 5\theta)^{\frac{2}{5}}\left(\cos\frac{2}{7}\theta + i\sin\frac{2}{7}\theta\right)^{2}}{(\cos 4\theta + i\sin 4\theta)^{\frac{1}{4}}\left(\cos\frac{2}{3}\theta - i\sin\frac{2}{3}\theta\right)^{3}}$$

$$= \frac{(\cos\theta + i\sin\theta)^{-5\times\frac{2}{5}}(\cos\theta + i\sin\theta)^{\frac{2}{7}\times 7}}{(\cos\theta + i\sin\theta)^{4\times\frac{1}{4}}(\cos\theta + i\sin\theta)^{-\frac{2}{3}\times 3}}$$

$$= \frac{(\cos\theta + i\sin\theta)^{-2}(\cos\theta + i\sin\theta)^{\frac{4}{7}}}{(\cos\theta + i\sin\theta)^{1}(\cos\theta + i\sin\theta)^{-2}}$$

$$= (\cos\theta + i\sin\theta)^{-2+\frac{4}{7}-1+2}$$

$$= (\cos\theta + i\sin\theta)^{-\frac{3}{7}}$$

$$= \cos\frac{3}{7}\theta - i\sin\frac{3}{7}\theta$$

Q.2(e) If $f(x) = x^2 - 4x + 11$, solve the equation $f(x) = f(3x - 1)$. **[4]**

(A) $f(x) = x^2 - 4x + 11$

$f(3x - 1) = (3x - 1)^2 - 4(3x - 1) + 11$

$= 9x^2 - 6x + 1 - 12x + 4 + 11$

$= 9x^2 - 18x + 16$

But $f(x) = f(3x - 1)$

$\therefore x^2 - 4x + 11 = 9x^2 - 18x + 16$

$\therefore -8x^2 + 14x - 5 = 0$ or $8x^2 - 14x + 5 = 0$

$\therefore x = \frac{5}{4}, \frac{1}{2}$ or 1.25, 0.5

Q.2(f) Simplify $i + i^{10} + i^{50} + i^{100}$. **[4]**

(A) $i + i^{10} + i^{50} + i^{100}$

$= 1 + (i^4)^{25} + (i^2)^5 + (i^2)^{25}$

$= 1 + (1)^{25} + (-1)^5 + (-1)^{25}$

$= 1 + 1 - 1 - 1$

$= 0$

Q.3 Attempt any FOUR of the following : **[16]**

Q.3(a) If $f(x) = ax^2 + bx + 3$ and $f(1) = 4$, $f(2) = 11$, find 'a' and 'b'. **[4]**

(A) $f(x) = ax^2 + bx + 3$

$\therefore f(1) = a(1)^2 + b(1) + 3$

$= a + b + 3$

$f(2) = a(2)^2 + b(2) + 3$

$= 4a + 2b + 3$

But $f(1) = 4$, $f(2) = 11$

$\therefore a + b + 3 = 4$

$4a + 2b + 3 = 11$

$\therefore a + b = 1$

$4a + 2b = 8$

$\therefore a = 3$

$b = -2$

Q.3(b) If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ then prove that $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$. **[4]**

(A) $$\therefore\ f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$$

$$= \log\left(\frac{1+x^2+2x}{1+x^2-2x}\right)$$

$$= \log\left[\frac{(1+x)^2}{(1-x)^2}\right]$$

$$= \log\left(\frac{1+x}{1-x}\right)^2$$

$$= 2\log\left(\frac{1+x}{1-x}\right)$$

$$= 2f(x)$$

Q.3(c) Evaluate $\lim\limits_{x\to1}\frac{x^3+3x^2-6x+2}{x^3+3x^2-3x-1}$. **[4]**

(A) $$\lim_{x\to1}\frac{x^3+3x^2-6x+2}{x^3+3x^2-3x-1} = \lim_{x\to1}\frac{(x-1)(x^2+4x-2)}{(x-1)(x^2+4x+1)}$$

$$= \lim_{x\to1}\frac{x^2+4x-2}{x^2+4x+1}$$

$$= \frac{(1)^2+4(1)-2}{(1)^2+4(1)+2}$$

$$= \frac{1}{2} \quad \text{or} \quad 0.5$$

Q.3(d) Evaluate $\lim\limits_{x\to\frac{\pi}{4}}\frac{\sin^2x-\cos^2x}{1-\tan x}$. **[4]**

(A) $$\lim_{x\to\frac{\pi}{4}}\frac{\sin^2x-\cos^2x}{1-\tan x} = \lim_{x\to\frac{\pi}{4}}\frac{(\sin x-\cos x)(\sin x+\cos x)}{1-\frac{\sin x}{\cos x}}$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{(\sin x-\cos x)(\sin x+\cos x)}{\frac{\cos x-\sin x}{\cos x}}$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{(\sin x-\cos x)(\sin x+\cos x)}{-(\sin x-\cos x)}\times\cos x$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{\sin x+\cos x}{-1}\times\cos x$$

$$= \frac{\sin\frac{\pi}{4}+\cos\frac{\pi}{4}}{-1}\times\cos\frac{\pi}{4}$$

$$= -1$$

Q.3(e) Evaluate $\lim\limits_{x\to\infty}\left(\sqrt{x^2+5x}-x\right)$. **[4]**

(A) $\lim\limits_{x\to\infty}\left(\sqrt{x^2+5x}-x\right)$

$= \lim\limits_{x\to\infty}\left(\sqrt{x^2+5x}-x\right)\times\dfrac{\sqrt{x^2+5x}+x}{\sqrt{x^2+5x}+x}$

$= \lim\limits_{x\to\infty}\dfrac{x^2+5x-x^2}{\sqrt{x^2+5x}+x}$

$= \lim\limits_{x\to\infty}\dfrac{5x}{\sqrt{x^2+5x}+x}$

$= \lim\limits_{x\to\infty}\dfrac{\frac{5x}{x}}{\frac{\sqrt{x^2+5x}+x}{x}}$

$= \lim\limits_{x\to\infty}\dfrac{5}{\sqrt{\frac{x^2+5x}{x^2}}+\frac{x}{x}}$

$= \lim\limits_{x\to\infty}\dfrac{5}{\sqrt{1+\frac{5}{x}}+1}$

$= \dfrac{5}{\sqrt{1+0}+1}$

$= \dfrac{5}{2}$ or 2.5

Q.3(f) Evaluate $\lim\limits_{x\to 0}\dfrac{6^x-3^x-2^x+1}{x^2}$. **[4]**

(A) $\lim\limits_{x\to 0}\dfrac{6^x-3^x-2^x+1}{x^2} = \lim\limits_{x\to 0}\dfrac{3^x\cdot 2^x-3^x-2^x+1}{x^2}$

$= \lim\limits_{x\to 0}\dfrac{3^x\left(2^x-1\right)-\left(2^x-1\right)}{x^2}$

$= \lim\limits_{x\to 0}\dfrac{\left(3^x-1\right)\left(2^x-1\right)}{x^2}$

$= \lim\limits_{x\to 0}\dfrac{\left(3^x-1\right)}{x}\times\dfrac{\left(2^x-1\right)}{x}$

$= \log 3\times\log 2$

Q.4Attempt any FOUR of the following : **[16]**

Q.4(a) If y = $\sin^{-1}$ (3x – 4x^3) find $\dfrac{dy}{dx}$. **[4]**

(A) Put $x = \sin\theta$

$\therefore y = \sin^{-1}(3x-4x^3)$

$= \sin^{-1}\left(3\sin\theta-4\sin^3\theta\right)$

$= \sin^{-1}(\sin 3\theta)$

$= 3\theta$

$= 3\sin^{-1}x$

$\therefore \dfrac{dy}{dx} = 3.\dfrac{1}{\sqrt{1-x^2}}$

Q.4(b) Using first principle find derivative of $f(x) = a^x$. **[4]**

(A) $f(x) = a^x$

$\therefore f(x + h) = a^{x+h}$

$$\therefore \frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{a^{x+h}-a^x}{h}$$

$$= \lim_{h\to 0}\frac{a^x\left(a^h-1\right)}{h}$$

$$= \lim_{h\to 0} a^x\left(\frac{a^h-1}{h}\right)$$

$$= a^x \log a$$

Q.4(c) If u and v are differentiable functions of x and y = u.v then prove that **[4]**

$$\frac{dy}{dx} = u\cdot\frac{dv}{dx}+v\cdot\frac{du}{dx}$$

(A) Let $y = uv$. Let δx be infinitesimal increment in x and δy, δu, δv be corresponding infinitesimal increments in y, u, v.

$\therefore y + \delta y = (u + \delta u)(v + \delta v)$

$= uv + u\delta v + v\delta u + \delta u\delta v$

$\therefore \delta y = uv + u\delta v + v\delta u + \delta u\delta v - y$

$= uv + u\delta v + v\delta u + \delta u\delta v - uv$

$= u\delta v + v\delta u + \delta u\delta v$

As δu and δv are very very small. $\delta u\delta v$ is negligible.

$\therefore \delta y = u\delta v + v\delta u$

$$\therefore \frac{\delta y}{\delta x} = \frac{u\delta v+v\delta u}{\delta x} = u\frac{\delta v}{\delta x}+v\frac{\delta u}{\delta x}$$

$$\therefore \lim_{\delta\to 0}\frac{\delta y}{\delta x} = \lim_{\delta x\to 0}\left[u\frac{\delta v}{\delta x}+v\frac{\delta u}{\delta x}\right]$$

$$\therefore \lim_{\delta x\to 0}\frac{\delta y}{\delta x} = u\lim_{\delta x\to 0}\frac{\delta v}{\delta x}+v\lim_{\delta x\to 0}\frac{\delta u}{\delta x}$$

$$\therefore \frac{dy}{dx} = u\frac{dv}{dx}+v\frac{du}{dx}$$

Q.4(d) Differentiate w.r.t x, $\tan^{-1}\left(\frac{5x}{1-6x^2}\right)$. **[4]**

(A) Let $y = \tan^{-1}\left(\frac{5x}{1-6x^2}\right)$

$$= \tan^{-1}\left(\frac{2x+3x}{1-2x\cdot 3x}\right)$$

Put $\tan A = 2x$ and $\tan B = 3x$

$$\therefore y = \tan^{-1}\left(\frac{\tan A+\tan B}{1-\tan A\ \tan B}\right)$$

$$= \tan^{-1}\left[\tan\left(A+B\right)\right]$$

$= A + B$

$= \tan^{-1}(2x) + \tan^{-1}(3x)$

$$\therefore \frac{dy}{dx} = \frac{1}{1+4x^2}\cdot 2+\frac{1}{1+9x^2}\cdot 3$$

Q.4(e) Find $\frac{dy}{dx}$ if $13x^2 + 2x^2y + y^3 = 1$. **[4]**

(A) $13x^2 + 2x^2 y + y^3 = 1$

$$\therefore\ 26x + 2\left(x^2\frac{dy}{dx} + y \cdot 2x\right) + 3y^2\frac{dy}{dx} = 0$$

$$\therefore\ 26x + 2x^2\frac{dy}{dx} + 4xy + 3y^2\frac{dy}{dx} = 0$$

$$\therefore\ 26x + 4xy + \left(2x^2 + 3y^2\right)\frac{dy}{dx} = 0$$

$$\therefore\ \frac{dy}{dx} = -\frac{26x + 4xy}{2x^2 + 3y^2}$$

Q.4 (f) If $y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$ and $x = \left(\sin^{-1}\left(\frac{2t}{1+t^2}\right)\right)$ find $\frac{dy}{dx}$. **[4]**

(A) $y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$

Put $t = \tan\theta$

$$\therefore\ y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = \tan^{-1}\left(\tan 2\theta\right) = 2\theta$$

$\therefore\ y = 2\tan^{-1}t$

And $x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$

Put $t = \tan\theta$

$$\therefore\ x = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$$

$\therefore\ x = 2\tan^{-1}t$

$\therefore\ y = x$

$\therefore\ \frac{dy}{dx} = 1$

Q.5 Attempt any FOUR of the following : **[16]**

Q.5(a) Evaluate $\lim_{x\to 1}\frac{\sin \pi x}{x-1}$. **[4]**

(A) Put $x - 1 = t$

$\therefore$ as $x \to 1, t \to 0$

$$\therefore\ \lim_{x\to 1}\frac{\sin\pi x}{x-1} = \lim_{t\to 0}\frac{\sin\pi(1+t)}{t}$$

$$= \lim_{t\to 0}\frac{\sin(\pi + \pi t)}{t}$$

$$= \lim_{t\to 0}\frac{-\sin\pi t}{t}$$

$$= -\lim_{t\to 0}\frac{\sin\pi t}{\pi t} \times \pi$$

$$= -1 \times \pi$$

$$= -\pi$$

Q.5(b) Evaluate $\lim_{x\to 3}\frac{\log x - \log 3}{x-3}$. **[4]**

(A) $\lim_{x\to 5}\left[\frac{\log x - \log 3}{x-3}\right]$

Let x = 3 + h or x–3 = h

as $x \to 3$, $h \to 0$

$$= \lim_{h\to 0}\left[\frac{\log(3+h)-\log 3}{3+h-3}\right]$$

$$= \lim_{h\to 0}\frac{1}{h}\log\left(\frac{3+h}{3}\right)$$

$$= \lim_{h\to 0}\log\left(1+\frac{h}{3}\right)^{1/h}$$

$$= \lim_{h\to 0}\log\left(1+\frac{h}{3}\right)^{3/h\times 1/3}$$

$$= \log e^{\frac{1}{3}}$$

$$= \frac{1}{3}\log e$$

$$= \frac{1}{3}$$

Q.5c) Using Bisection method find the approximate root of $x^2 + x - 3 = 0$ (3 iterations). **[4]**

(A) $x^2 + x - 3 = 0$

$f(x) = x^2 + x - 3$

$\therefore$ f(1) = –1

f(2) = 3

$\therefore$ the root is in (1, 2).

$\therefore x_1 = \frac{1+2}{2} = 1.5$

$\therefore$ f(1.5) = 0.75

$\therefore$ the root is in (1, 1.5).

$\therefore x_2 = \frac{1+1.5}{2} = 1.25$

$\therefore$ f(1.25) = –0.188

$\therefore$ the root is in (1.25, 1.5)

$\therefore x_3 = \frac{1.25+1.5}{2} = 1.375$

Q.5(d) Using False Position method find the root of $x^3 - x - 4 = 0$ (3 iterations only). **[4]**

(A) $f(x) = x^3 - x - 4$

$\therefore$ f(1) = –4

f(2) = 2

$\therefore$ the root is in (1, 2)

$\therefore x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)} = 1.667$

$\therefore$ f(1.667) = –1.035

$\therefore$ the root is in (1.667, 2).

$\therefore x_2 = 1.781$

$\therefore$ f(1.781) = –0.132

$\therefore$ the root is in (1.781, 2).

$\therefore x_3 = 1.795$

Q.5(e) Using Newton–Raphson method find the root of x^4 – x – 9 = 0 (carry out 3 iterations). **[4]**

(A) $x^4 - x - 9 = 0$

$\therefore f(x) = x^4 - x - 9$

$\therefore f'(x) = 4x^3 - 1$

$\therefore f(1) = -9$

$f(2) = 5$

$$x - \frac{f(x)}{f'(x)} = x - \frac{x^4 - x - 9}{4x^3 - 1}$$

$$= \frac{3x^4 + 9}{4x^3 - 1}$$

OR

$$\frac{xf'(x) - [f(x)]}{f'(x)} = \frac{x(4x^3 - 1) - (x^4 - x - 9)}{4x^3 - 1}$$

$$= \frac{3x^4 + 9}{4x^3 - 1}$$

Start with $x_0 = 2$,

$\therefore x_1 = 1.839$

$x_2 = 1.814$

$x_3 = 1.813$

Q.5(f) Using Newton–Raphson method find approximate value of $\sqrt{10}$ (3 iterations). **[4]**

(A) Let $x = \sqrt{10}$

$\therefore x^2 - 10 = 0$

$\therefore f(x) = x^2 - 10$

$\therefore f'(x) = 2x$

$\therefore f(3) = -1$

$f(4) = 6$

$$x - \frac{f(x)}{f'(x)} = x - \frac{x^2 - 10}{2x} \quad \text{...(i)}$$

$$= \frac{x^2 + 10}{2x} \quad \text{...(ii)}$$

OR

$$\frac{xf'(x) - [f(x)]}{f'(x)} = \frac{x(2x) - (x^2 - 10)}{2x} \quad \text{...(i)}$$

$$= \frac{x^2 + 10}{2x} \quad \text{...(ii)}$$

Start with $x_0 = 3$,

$\therefore x_1 = 3.167$

$x_2 = 3.162$

$x_3 = 3.162$

Q.6Attempt any FOUR of the following : **[16]**

Q.6(a) If y = sin 5x – 3 cos 5x show that $\frac{d^2y}{dx^2}$ + 25y = 0. **[4]**

(A) y = sin 5x – 3cos5x

$\therefore \frac{dy}{dx} = \cos 5x \cdot 5 + 3\sin 5x \cdot 5$

$= 5\cos 5x + 15\sin 5x$

$\therefore \dfrac{d^2y}{dx^2} = -25\sin 5x + 75\cos 5x$

$= -25(\sin 5x - 3\cos 5x)$

$= -25y$

$\therefore \dfrac{d^2y}{dx^2} + 25y = 0$

Q.6(b) If $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ at $\theta = \dfrac{\pi}{4}$. [4]

(A) $x = a(\theta - \sin\theta)$

$\therefore \dfrac{dx}{d\theta} = a(1 - \cos\theta)$

$y = a(1 - \cos\theta)$

$\dfrac{dy}{d\theta} = a(\sin\theta)$

$\therefore \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{a(\sin\theta)}{a(1-\cos\theta)} = \dfrac{\sin\theta}{1-\cos\theta}$

$\therefore$ at $\theta = \dfrac{\pi}{4}$, $\dfrac{dy}{dx} = \dfrac{\sin\frac{\pi}{4}}{1-\cos\frac{\pi}{4}}$

$= \dfrac{1}{\sqrt{2}-1}$ or 1.793

Q.6(c) Solve by Jacobi's method performing 3 iterations : [4]

$20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$

(A) $20x + y - 2z = 17$

$3x + 20y - z = -18$

$2x - 3y + 20z = 25$

$\therefore x = \dfrac{1}{20}(17 - y + 2z)$

$y = \dfrac{1}{20}(-18 - 3x + z)$

$z = \dfrac{1}{20}(25 - 2x + 3y)$

Starting with $x_0 = 0$ $y_0 = z_0$

$x_1 = 0.85$

$y_1 = -0.9$

$z_1 = 1.25$

$x_2 = 1.02$

$y_2 = -0.965$

$z_2 = 1.03$

$x_3 = 1.001$

$y_3 = -1.002$

$z_3 = 1.003$

Q.6(d) Solve by Gauss-Seidal method (3 iterations) **[4]**

$15x + 2y + z = 18$, $2x + 20y - 3z = 19$, $3x - 6y + 25z = 22$

(A) $\therefore$ $x = \frac{1}{15}(18 - 2y - z)$

$y = \frac{1}{20}(19 - 2x + 3z)$

$z = \frac{1}{25}(22 - 3x + 6y)$

Starting with $x_0 = 0 = y_0 = z_0$

$x_1 = 1.2$

$y_1 = 0.83$

$z_1 = 0.935$

$x_2 = 1.027$

$y_2 = 0.988$

$z_2 = 0.994$

$x_3 = 1.002$

$y_3 = 0.999$

$z_3 = 0.999$

Q.6(e) Solve by Gauss Elimination method : **[4]**

$x + 2y + 3z = 14, 3x + y + 2z = 11, 2x + 3y + z = 11$

(A) $x + 2y + 3z = 14$

$3x + y + 2z = 11$

$2x + 3y + z = 11$

$$\begin{array}{r} 3x + 6y + 9z = 42 \\ 3x + y + 2z = 11 \\ - \quad - \quad - \quad - \\ \hline 5y + 7z = 31 \end{array} \quad \text{and} \quad \begin{array}{r} 6x + 2y + 4z = 22 \\ 6x + 9y + 3z = 33 \\ - \quad - \quad - \quad - \\ \hline -7y + z = -11 \end{array}$$

$$\begin{array}{r} 5y + 7z = 31 \\ -49y + 7z = -77 \\ + \quad - \quad - \\ \hline 54y = 108 \end{array}$$

$\therefore$ $y = 2$

$z = 3$

$x = 1$

Q.6(f) Solve by Gauss–Seidal method (2 iterations) **[4]**

$5x - y = 9$, $x - 5y + z = -4$, $y - 5z = 6$, Taking $x_0 = 1.5$, $y_0 = 0.5$, $z_0 = -0.5$

(A) $5x - y = 9$ $\quad$ $5x - y + 0z = 9$

$x - 5y + z = -4$ $\quad$ OR $\quad$ $x - 5y + z = -4$

$y - 5z = 6$ $\quad$ $0x + y - 5z = 6$

$\therefore$ $x = \frac{1}{5}(9 + y)$

$y = \frac{1}{-5}(-4 - x - z)$

$z = \frac{1}{-5}(6 - y)$

Starting with $x_0 = 1.5$, $y_0 = 0.5$, $z_0 = -0.5$

$x_1 = 1.9$

$y_1 = 1.08$

$z_1 = -0.984$

$x_2 = 2.016$

$y_2 = 1.006$

$z_2 = -0.999$

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