Engineering Mechanics

Time : 3 Hrs.] Prelim Question Paper Solution [Marks : 100

Q.1 Attempt any TEN of the following : **[20]**

Q.1(a) Difference between mass and weight? **[2]**

(A) Difference between mass and weight

	Mass	Weight
1.	Mass is the quantity of matter contained in a body.	Weight of a body is the force with which the body is attracted by the earth towards its centre.
2.	It is scalar quantity.	it is a vector quantity.
3.	S.I. unit of mass is 'ky'	S.I. hit of weight is 'N'.

Q.1(b) State Newton's Laws of motion? (First, Second & Third) **[2]**

(A) **Nowtons first law :** " Everybody continues to be in its state of rest or of uniform motion in a straight line, unless it is acted upon by some external agency".

Second Law : "The rate of change of momentum is directly proportional to applied forces".

Third Law : To every action there is equal and opposite reaction.

Q.1(c) Define Resolution of force. **[2]**

(A) **Resolution of force :**

The way of representing a single force into number of forces without changing the effect of force on body is called resolution of force.

Q.1(d) Define effort and effort lost in friction. **[2]**

(A) **Effort(P) :** The force applied to lift the heavy loads is known as effort.

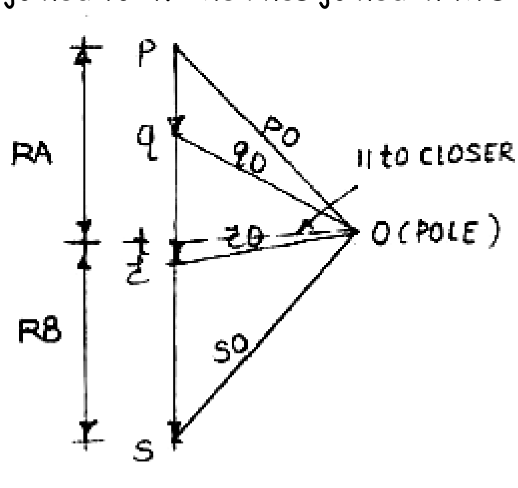
Effort lost in friction (P_f): It is the effort by considering the wear and tear effect while use of machine.

OR

It is the effort obtained by subtracting ideal effort from an effort.

Q.1(e) What is polar diagram? **[2]**

(A) **Polar Diagram :** In case of non-concurrent or parallel force system the point of application of resultant can be found out by constructing polar diagram. Polar diagram is obtained from the vector diagram. To construct a polar diagram, any point "O" known as pole is chosen near the vector diagram and the points on the vector diagram are joined to it. The lines joined in this way are known as rays.



Q.1(f) State types of friction. **[2]**

(A) **(i) Static friction :** The friction experienced by a body when it is in equilibrium.

(ii) Dynamic friction : The friction experienced by a body when it is in motion.

(iii) Rolling : The friction experienced by a bodies when one rolls over the another body.

(iv) Sliding : The friction experienced by a bodies when one slides over the another body.

Q.1(g) State principle of transmissibility of force. **[2]**

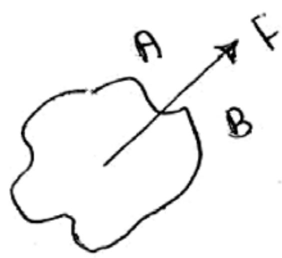
(A) **Principle of transmissibility of force :**

If force acts at a point on a rigid body, it is assumed to act at any other point on line of action of force within the body.

Q.1(h) What is Bow's notation ? Explain with a sketch. **[2]**

(A) **Bow's notation**

Bow's notation is used designate a force as per this notation, each force is designated or named by two spaces one on each side of the line of action of a force. This space are generally named by capital letter's as A, B, C … serially.



Explanation

A force say 'F' acting on rigid body divided space above or below it into two parts, say A and B hence the force 'F' is named as AB.

Q.1(i) Explain meaning of self locking machine. State the condition for it. **[2]**

(A) **Self–Locking Machine**

A machine which is not capable of doing work in reverse direction even on removal of effort, then the machine is called as self–locking or Non–reversible machine.

Condition for Self–Locking Machine

Efficiency < 50 %

$\therefore \quad \eta < 50\ \%$

Q.1(j) State velocity ratio for screw jack with meaning of term involved. **[2]**

(A) Velocity Ratio of Simple Screw jack is given by

VR = $2\pi L / P$ –– When handle of length L is provided

OR

VR = $2\pi R / P$ –– When effort wheel is provided

Where, L = length of handle

P = pitch of screw

R = radius of an effort wheel.

Q.1(k) What is coefficient of friction? **[2]**

(A) **Coefficient of friction :**

Coefficient of friction is defined as the ratio of limiting friction to the normal reaction at the surfaces of contact.

$$\mu = F/R$$

Q.1(l) What is efficiency of a machine? **[2]**

(A) Efficiency (η) : The efficiency of a machine is the ratio of output to the input of a machine and is generally expressed as a percentage.

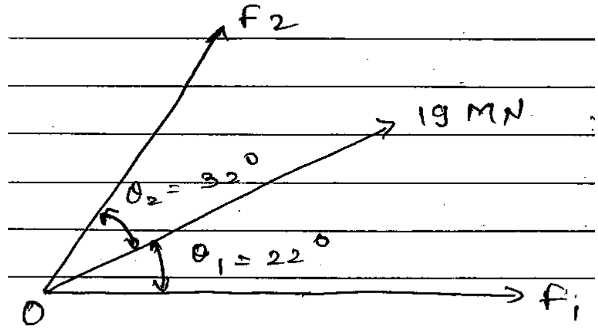
$$\%\eta = \frac{\text{Output}}{\text{input}} \times 100$$

Q.2 Attempt any FOUR of the following : **[16]**

Q.2(a) Resolve the force 19 MN along 22° and 32° on either side of it. **[4]**

(A) Resolve the force 19 MN along 22° and 32° on either side of it.

Let $\theta_1 = 22°$ and $\theta_2 = 32°$.



Referring figure,

$$F_1 = \frac{F\sin\theta_2}{\sin(\theta_1+\theta_2)} = \frac{19\sin 32°}{\sin(22°+32°)}$$

$F_1 = 12.44$ MN

$$F_2 = \frac{F\sin\theta_1}{\sin(\theta_1+\theta_2)} = \frac{19\sin 22°}{\sin(22°+32°)}$$

$\therefore$ $F_2 = 8.79$ MN.

Q.2(b) In a simple axle and wheel, the diameter of wheel is 180 mm and that of axle 30 mm. If the efficiency of the machine is 80%, find the effort required to lift a load of 100 N. **[4]**

(A) **Given:**

D = diameter of wheel = 180 mm,

d = diameter of axle = 30 mm

η = efficiency of the machine = 80%

W = load = 100 N

To find : V.R: ?

Formula: $V.R = \dfrac{D}{d}$

Solution:–

$$V.R. = \frac{D}{d} = \frac{180}{30} = 6$$

Effort (p) required to lift a load of 100N:

$$\eta = \frac{M.A}{V.R.}\times 100$$

$$\eta = \frac{W}{P\times V.R.}\times 100$$

$$\eta = \frac{W.A}{V.R.}\times 100 \qquad \because\ MA = \frac{W}{P}$$

$$80 = \frac{100}{P\times 6}\times 100$$

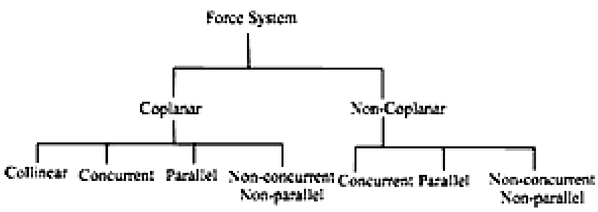
$$\therefore\ P = \frac{100\times 100}{80\times 6}$$

$= 20.83$ N

Q.2(c) Write the different types of force system. **[4]**

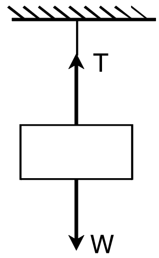
(A) **Classification of force system :**

Based on Line of Action, Force system may be classified as following :



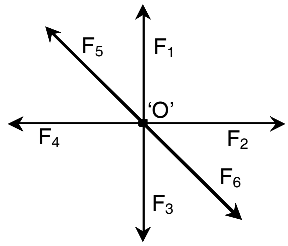
(i) Collinear forces System : The forces acting in same line of action is called collinear forces. A collinear system is necessarily coplanar

Weight W and tension T are in same line of action.



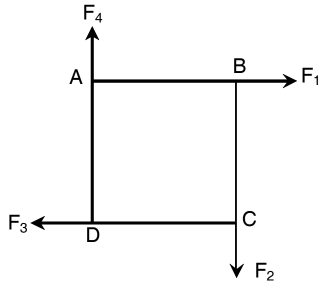
(ii) Concurrent forces : The system in which all the forces act at same point is called as concurrent forces. A concurrent force system may be either coplanar or non-coplanar provided that there are more than two forces.

All the forces F_1, F_2, F_3, F_4, F_5, F_6 are meeting at point 'O'.



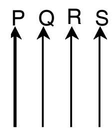
(iii) Non–concurrent forces : The system in which the forces act at different points is called non–concurrent forces. A non-concurrent system may be either coplanar or non-coplanar

In the diagram F_1, F_2, F_3, F_4 are acting at different point.



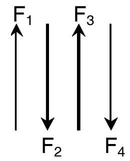
Parallel forces

The system of in which line of action are parallel to each other are called as parallel forces. A parallel force system may be either coplanar or non-coplanar.



(i) Like parallel forces :
Like forces acting in same direction are called as like parallel forces.

(ii) Unlike parallel forces :
Parallel forces acting in opposite direction are called as unlike parallel forces.



Q.2(d) A screw jack of pitch 8 mm a lever of 250 mm length if the efficiency of machine is 30%, find the effort required to lift a load of 1500 N. **[4]**

(A) Pitch = 8 mm, L = 250 mm, η = 30 %.
W = 1500N
find effort (P) = ?

$$V.R = \frac{2\pi L}{P} = \frac{2\pi \times 250}{8} = 196.35$$

$$M.A. = W/P = \frac{1500}{P}$$

$$\eta = \frac{M.A}{V.R}\times 100 \quad \Rightarrow \quad 30 = \frac{1500}{P}\times\frac{1}{196.35}\times 100$$

$\therefore$ P = 25.46N

Q.2(e) For a general pulley block number of cogs on effort wheel is 24, that of on load wheel is 6 No. of teeth on the pinion is 4 and that of on spur is 36. If the maximum effort, which can be applied is 60 N, calculate the maximum load that can be lifted, if efficiency of machine is 80%. **[4]**

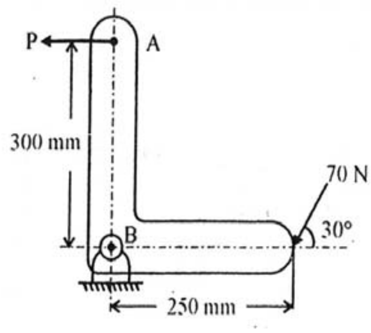
(A) Given :
Geared pulley block machine
No of cogs on effort wheel (N_1) = 24
No of cogs on load wheel (N4) = 6
No of cogs on pinion (N2) = 4

No of cogs on spur (N3) = 36
Max effort (P) = 60 N,
(η) = 80%
Find : Max load lifted by machine (W)
Solution :

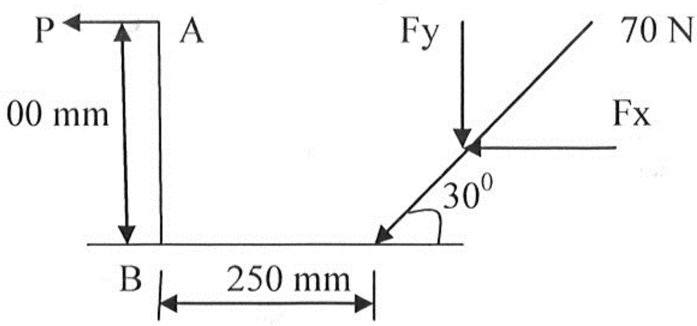
(i) for given machine VR is given by
V. R. = N_1 X N_3 / N_2 X N_4 = 24 $\times$ 36 / 4 $\times$ 6 = 36
VR = 36

(ii) Efficiency (η) = MA / VR X 100
80 = MA / 36 $\times$ 100
MA = 28.8
But, MA = W/P
28.8 = W/60
W = 1728 N

Q.2(f) A crank ABC with system of forces acting on it is shown in figure. Find force "P' to maintain equilibrium. **[4]**



(A) Given : 70 N force acting at 30° inclination as shown
Find : P, if equilibrium is maintained

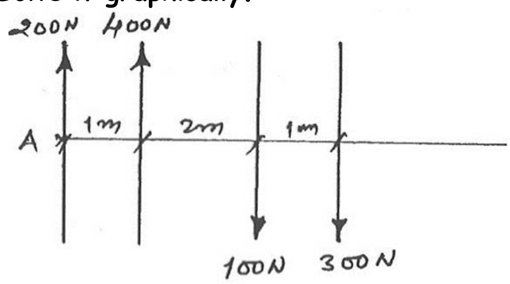


Taking moment @ point B and considering equilibrium condition

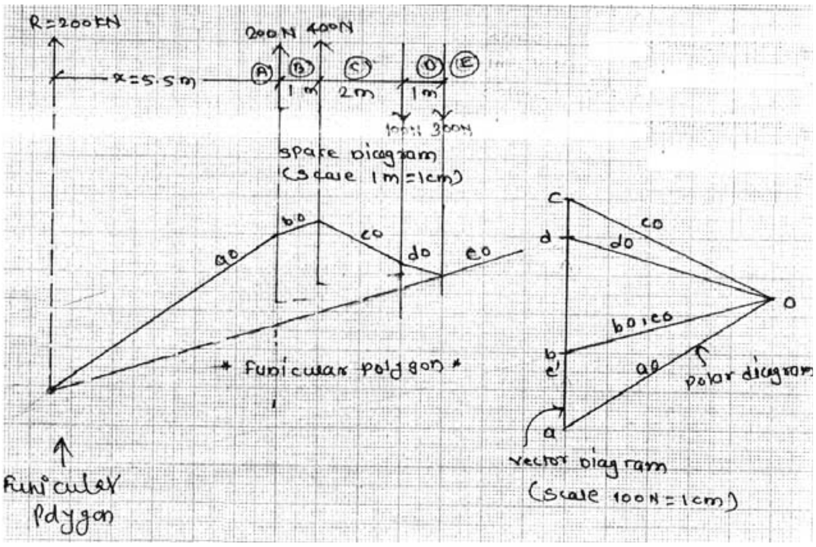
$\sum M_B$ = 0
= Fy $\times$ 250 – P $\times$ 300 = 70 sin 30 $\times$ 250 = 300P
P = 29.17 N

Q.3 Attempt any FOUR of the following : **[16]**

Q.3(a) Determine the magnitude of resultant and position of it wrt point A for the force system shown in Figure. Solve it graphically. **[4]**

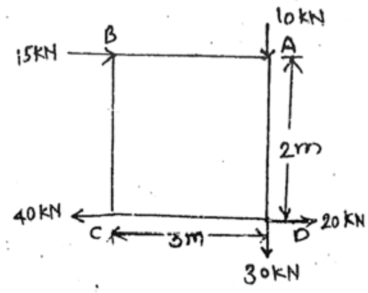


(A)



Resultant (R) = $\ell(ae) \times$ scale
= 2×100 = 200 kN
x = 5.5×1 = 5.5 m
Position (x) = 5.5 m

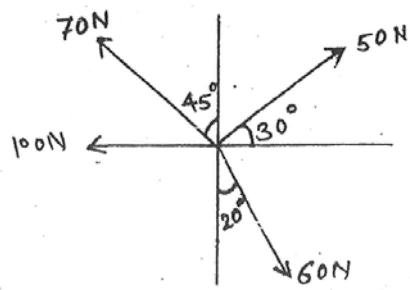
Q.3(b) Calculate the moment about point 'B' for the force system as shown in Figure. **[4]**



(A) Taking moment @ point B –

M_B = (15 x 0) + (10 x 3) – (20 x 2) + (30 x 3) + (40 x 2)
= 0 + 30 – 40 + 90 + 80
= +160 N- m (↻)
= 160 N-m (Clockwise moment)

Q.3(c) Calculate the magnitude and direction of resultant for concurrent force system as shown in Figure. Use analytical method. **[4]**



(A) (1) Resolving all forces

$\sum Fx$ = +(50 cos 30) – (70 cos 45) + (100 cos 180) + (60 cos 70)
= + 43.30 – 49.50 – 100 + 20.52
= - 85.68

$\sum Fy$ = +(50 sin 30) + (70 sin 45) + (100 sin 180) – (60 sin 70)
= + 25 + 49.50 + 0 – 56.38

= + 18.12 N

(2) Magnitude of Resultant

$$R = \sqrt{\left(\sum Fx\right)^2 + \left(\sum Fy\right)^2}$$

$$R = \sqrt{(-85.68)^2 + (18.12)^2}$$

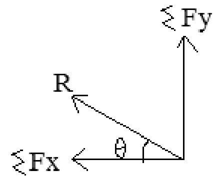
$$R = 87.58N$$

(3) Direction and position of resultant

As $\sum$ Fx is –ve and $\sum$ Fy is +ve, resultant lies in 2nd quadrant.

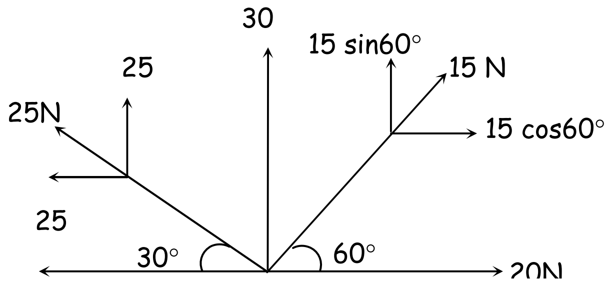
$$\theta = \tan^{-1}\left|\frac{\sum Fy}{\sum Fx}\right| = \tan^{-1}\left|\frac{18.12}{85.68}\right|$$

$$\theta = 11.94°$$



Q.3(d) Four forces 20N, 15N, 30N, & 25N are acting at 0°, 60°, 90° & 150° from x–axis taken in order. Find resultant by analytical method. **[4]**

(A)



$$\Sigma Fx = 20 + 15\cos 60° - 25 \cos 30°$$

$$\Sigma F_X = 5.85 \text{ N}$$

$$\Sigma F_y = 15 \sin 60° + 30 + 25 \sin 30°$$

$$\Sigma F_y = 55.49 \text{ N}$$

Resultant force is given by,

$$R = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(5.85)^2 + (55.49)^2}$$

$$R = 55.79 \text{ N}$$

Direction:

$$\theta = \tan^{-1}\left(\frac{F_X}{F_y}\right) = \tan^{-1}\left(\frac{55\cdot 49}{5.85}\right)$$

θ = 83.98° with horizontal.

Q.3(e) Find the angle between two forces of magnitude 120 N each, such that their resultant is 60 N. **[4]**

(A) Given :

P = Q = 120 N

R = 60 N

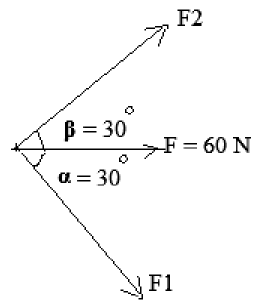
To find :

θ

Solution :

Using Law of parallelogram of forces

$R_2 = P_2 + Q_2 + 2PQ \cos \theta$
$(60)^2 = (120)^2 + (120)^2 + 2X120X120\cos\theta$
$3600 = 14400 + 14400 + 28800 \cos\theta$
$3600 = 28800 + 28800 \cos\theta$
$3600 - 28800 = 28800 \cos\theta$
$- 25200 = 28800 \cos\theta$
$\theta = \cos^{-1}\left(\frac{-25200}{28800}\right)$
$\theta = \cos^{-1}(-0.875)$
$\theta = 151.04$



Q.3(f) What are the components of 60 N force acting horizontal, in two directions on either side at an angle of 30° each? **[4]**

(A) $F1 = \frac{F\sin\alpha}{\sin(\alpha+\beta)} = \frac{60\sin 30}{\sin(30+30)} = 34.64N$

$F2 = \frac{F\sin\beta}{\sin(\alpha+\beta)} = \frac{60\sin 30}{\sin(30+30)} = 34.64N$

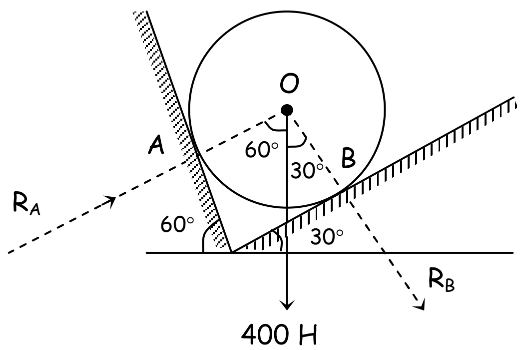
Q.4 Attempt any FOUR of the following : **[16]**

Q.4(a) A sphere of weight 400 N rests in a groove of smooth inclined surfaces which are making 60° and 30° inclination to the horizontal. Find the reactions at the contact surfaces. **[4]**

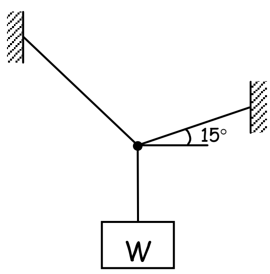
(A) Apply Lami's theorem :

$$\frac{R_A}{\sin 30^\circ} = \frac{R_B}{\sin 60^\circ} = \frac{400}{\sin(60^\circ+30^\circ)} = \frac{40}{\sin 90^\circ}$$

$R_A = 0.5 \times 400 = 200N$
$R_B = 0.866 \times 400 = 346.4N$

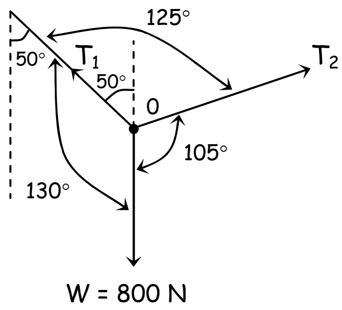


Q.4(b) Check whether a wire having capacity of 600 N can lift a load of 800N if it is attached as shown in Figure. **[4]**



W = 800 N

(A) Apply Lami's theorem :



$$\frac{T_1}{\sin 105^\circ} = \frac{T_2}{\sin 130^\circ} = \frac{800}{\sin 125^\circ}$$

$$\frac{T_1}{0.966} = \frac{T_2}{0.766} = 976.62$$

taking, $\frac{T_1}{0.966} = 976.62$

$T_1 = 0.966 \times 976.62 = 943.34\text{N} > 800\text{N}$

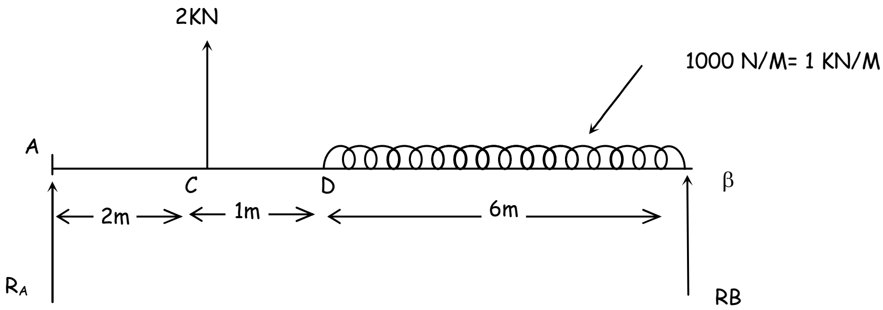
taking $\frac{T_2}{0.760} = 976.62$

$T_2 = 0.766 \times 976.62 \quad = 748.1\text{N} < 800\text{N}$

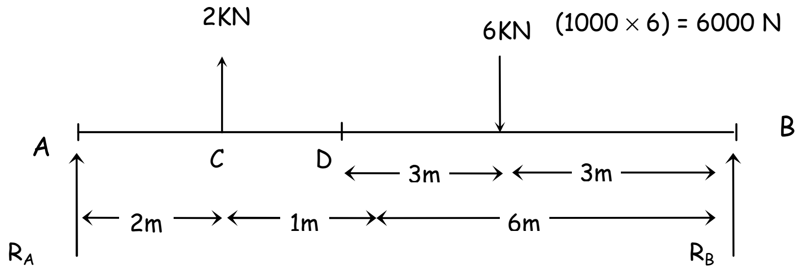
Since wire has a capacity of 800N. But tension in one part is 943.34N, it cannot lift a load of 800N

Q.4(c) A beam AB of 9m span is simply supported at ends. The beam carries point load of 2KN upwards at 2m from A and uniformly distributed load of 1000 N/m downwards on a length of 6m from B. Determine support reactions analytically. **[4]**

(A)



Converting u.d. l to its equivalent load;



taking moments about A,

$\Sigma M_A = 0$

$(R_A \times 0) - (2 \times 2) + (6 \times 6) - (R_B \times 9) = 0$

$-4 + 36 - 9R_B = 0$

$9R_B = 32$

$R_B = 3.55\text{KN}$

also,

$\Sigma F_y = 0$

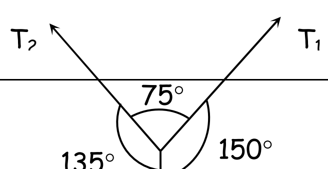
$R_A + R_B + 2 - 6 = 0$

$R_A + R_B = 4$

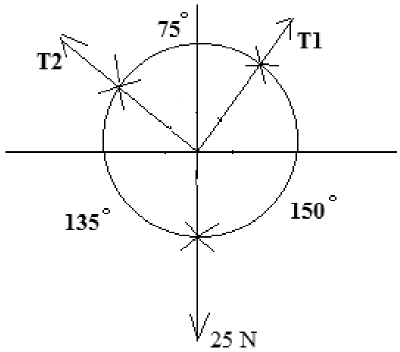
$R_A = 4 - 3.55$

$R_A = 0.45$ KN.

Q.4(d) Find the tensions in the string as shown in figure. **[4]**

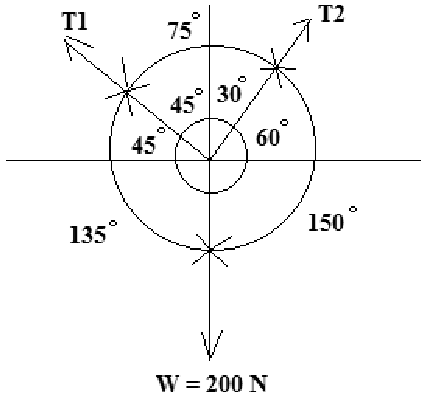


(A)



Using Lami's theorem,

$$\underset{(1)}{\frac{25}{\sin 75}} = \underset{(2)}{\frac{T_1}{\sin 135}} = \underset{(3)}{\frac{T_2}{\sin 150}}$$



Using term (1) and (2)

$$\frac{25}{\sin 75} = \frac{T_1}{\sin 135}$$

$$T_1 = \frac{\sin 135}{\sin 75} X 25$$

T_1 = 18.301N

Using term (1) and (3)

$$\frac{25}{\sin 75} = \frac{T_2}{\sin 150}$$

$$T_2 = \frac{\sin 150}{\sin 75} X 25$$

T_2 = 13.940N

Q.4(e) Two men carry a weight 200 N by means of ropes fixed to the weight. One rope is inclined at 45° and other 30° with the vertical. Find tension in each side of rope. **[4]**

(A) Using Lami's theorem,

$$\frac{W}{\sin 75^\circ} = \frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 135^\circ}$$

$$\underset{(1)}{\frac{200}{\sin 75^\circ}} = \underset{(2)}{\frac{T}{\sin 150^\circ}} = \underset{(3)}{\frac{T_2}{\sin 135^\circ}}$$

Using term (1) and (2)

$$\frac{200}{\sin 75} = \frac{T_1}{\sin 150°}$$

$T_1 = 200 \times \dfrac{\sin 150°}{\sin 75°}$

$T_1 = 103.527$ N

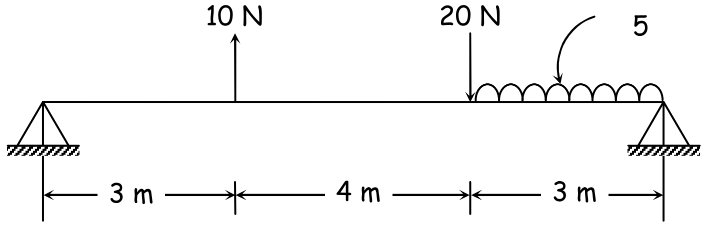
Using term (1) and (3)

$$\frac{200}{\sin 75} = \frac{T_2}{\sin 135°}$$

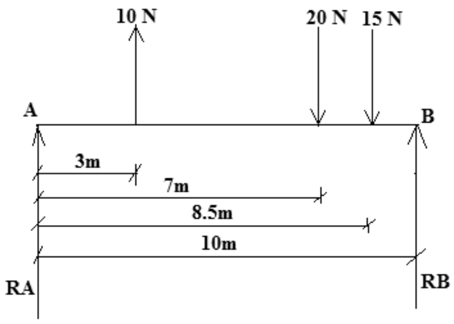
$T_2 = 200 \times \dfrac{\sin 135°}{\sin 75°}$

$T_2 = 146.410$ N

Q.4(f) Find the support reactions of simply supported beam shown in figure. **[4]**



(A)



(i) Equivalent point load and it's position

Equivalent point load = Intensity of udl

$= 5 \times 3$

$= 15$ N

Position from RA= 7m + Span of udl / 2 = 7 + (3/2) = 8.5 m

(ii) Applying equilibrium conditions

$\sum Fy = 0$ ($\uparrow$ +ve, $\downarrow$ –ve) and $\sum M = 0$ ($\uparrow$ +ve, $\uparrow$ – ve)

$\sum Fy = 0$

RA + 10 – 20 – 15 + RB = 0

RA + RB = 25 N ... (1)

$\sum M_A = 0$

Taking moment of all forces @ point A

$(RA \times 0) - (10 \times 3) + (20 \times 7) + (15 \times 8.5) - (RB \times 10) = 0$

RB = 23.75 N

Putting value of RB in equation (1)

RA + 23.75 = 25

RA = 1.25 N

Q.5 Attempt any FOUR of the following : **[16]**

Q.5(a) A Ladder of weight 400N and length 10m is supported on smooth well with its lower end 4m from the wall. The coefficient of friction between the flower and the ladder is **[4]**

0.3. Show the forces acting on the ladder and find frictional force at floor.

(A) $\sin\theta = 4/10$

$\theta = \sin^{-1}(4/10)$

$\theta = 23.58°$

$\Sigma Fx = 0$

$R_W - E_F = 0$

$R_W = F_F$

$\Sigma Fy = 0$

$F_W + R_F - W = 0$

$0 + R_F - 400 = 0$

$R_F = 400$ N

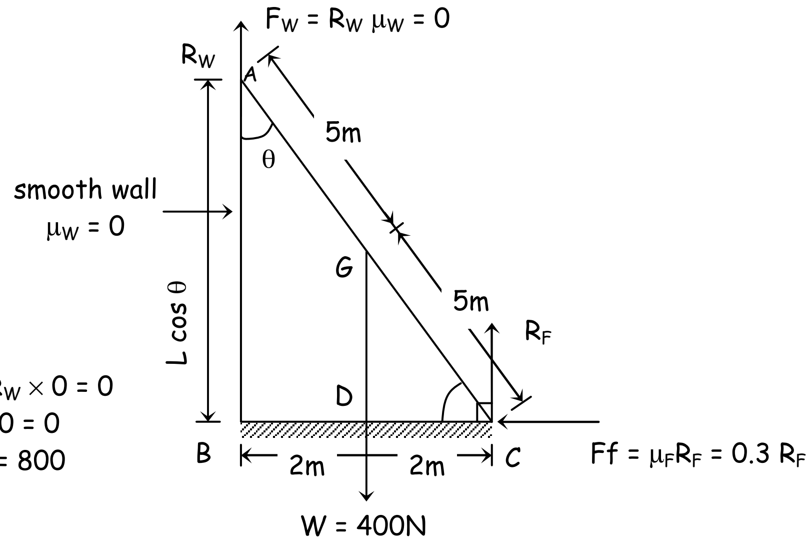
$\Sigma MA = 0$

$F_F \times L\cos\theta - R_F \times 4 + 400 \times 2 + R_W \times 0 = 0$

$F_F \times 10\cos 23.58° - 400 \times 4 + 800 = 0$

$F_F \times 10\cos 23.58° = 1600 - 800 = 800$

$$F_F = \frac{800}{10\cos 23.58} = 87.25 \text{ N}$$



Ans. :

(i) $R_W = F_F = 87.29$N (ii) $R_F = 400$ N (iii) $F_W = 0$

Q.5(b) For a certain machine an effort of 100 N and 150 N can lift a load of 1 kN and 2kN respectively. Find the law of machine. Also calculate maximum efficiency if VR is 20. **[4]**

(A) Effort (P) = 100 N and W = 1 kN = 1000 N

Effort (P) = 150 N and W = 2 kN = 2000 N

(i) Law of machine

$P = mW + C$

$100 = m \times 1000 + C$ … (1)

$150 = m \times 2000 + C$ … (2)

Multiplying equation (1) by 2

$\therefore$ $1000\,m + C = 100 \times 2$

$2000\,m + C = 150$

$\therefore$ Subtracting equation (2) from equation (1)

$2000\,m + 2C = 200$ … (1)

$2000\,m + C = 150$ … (2)

$-\qquad\qquad -$

$C = 50$ N put in equation (2)

$150 = m \times 2000 + 50$

$150 - 50 = 2000\,m$

$100 = 2000\,m$

$m = 0.05$

(ii) Law of machine

$P = (0.05W + 50)$ N

(iii) Max.M.A. $= \dfrac{1}{m} = \dfrac{1}{0.05}$

Max M.A. = 20

(iv) Max. η = $\frac{Max.M.A.}{V.R.} \times 100 = \frac{20}{20} \times 100$

Max. η = 100 %

Q.5(c) A block of weight 500 N is placed on a inclined plane at an angle of 20° with the horizontal. If coefficient of friction is 0.14, find the force 'P' applied Parallel to the plane, just to move the body up the plane. **[4]**

(A) $\Sigma Fy = 0$

$R - 500 \cos 20° = 0$

$R = 469.85$ N

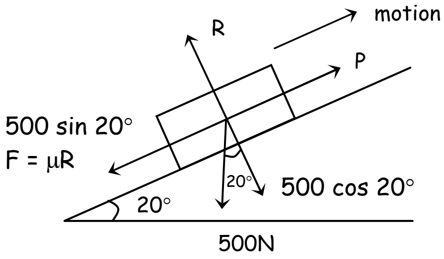
$\Sigma Fx = 0$

$P - 500 \sin 20° - \mu R = 0$

$P = 500 \sin 20° + \mu R$

$= 171.01 + 0.14 \times 469.85$

$P = 236.789$N



Q.5(d) The velocity ratio of a certain machine is 72. The law of machine is $P = \left(\frac{1}{48}W + 30\right)$N. Find the maximum mechanical advantage and maximum efficiency. State also whether the machine is reversible or not. **[4]**

(A) VR = 72

$P = \left(\frac{1}{48}W + 30\right)N$

$\therefore$ $m = \frac{1}{48}$ and c = 30N

(i) Maximum M.A. = ?

(ii) Maximum η = ?

(iii) To decide whether the machine is reversible or not.

(i) Maximum M.A. = $\frac{1}{m} = \frac{1}{1/48} = 48$

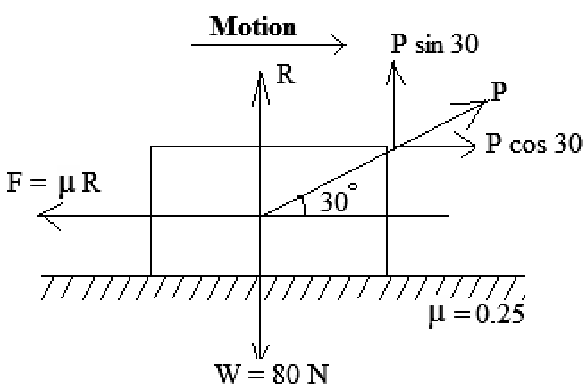
(ii) Maximum η = $\frac{1}{m \times V.R.} \times 100$

$= \frac{1}{1/48 \times 72} \times 100 = 66.67\% > 50\%$

(iii) Since the maximum efficiency is more than 50%, the machine is reversible.

Q.5(e) A block of 80 N is placed on a horizontal plane where the coefficient of friction is 0.25. Find the force at 30° up the horizontal to just move the block **[4]**

(A)



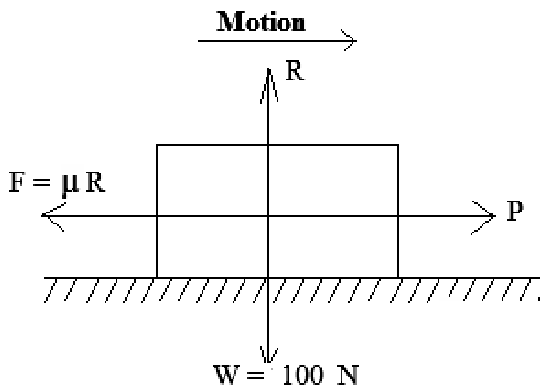
For limiting equilibrium | ($\rightarrow$ +ve, $\leftarrow$ −ve)

($\uparrow$ +ve, $\downarrow$ –ve)
$\sum Fy = 0$
$+ R - W = 0$
$R = W = 2000$ N
$R = 2000$ N

$\sum Fx = 0$
$+ P - F = 0$
$+ P = F$
$P = \mu R$ ———— Since $F = \mu R$
$P = 0.4 \times 2000$
$P = 800$ N

Q.5(f) Find the horizontal force required to drag a body of weight 100 N along a horizontal plane. If the plane is raised gradually upto 15°, the body will begin to slide. **[4]**

(A)



We know, $\mu = \tan \alpha = \tan 15° = 0.27$

For limiting equilibrium
($\uparrow$ +ve, $\downarrow$ –ve)
$\sum Fy = 0$
$+ R - W = 0$
$R = W = 100$ N
$R = 100$ N

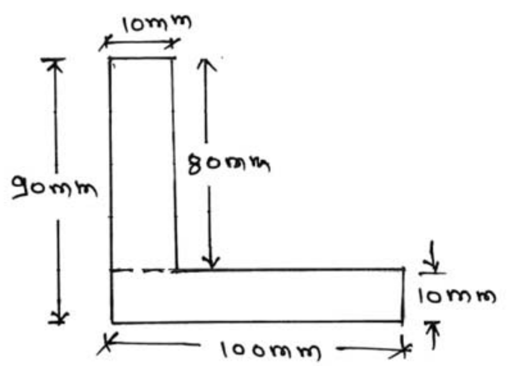
($\rightarrow$ +ve, $\leftarrow$ –ve)

$\sum Fx = 0$
$+ P - F = 0$
$+ P = F$
$P = \mu R$ ———— Since $F = \mu R$
$P = 0.27 \times 100$
$P = 27$ N

Q.6 Attempt any FOUR of the following : **[16]**

Q.6(a) Locate the centroid of angle section 90 mm × 100 mm × 10 mm. (90 mm side is vertical.) **[4]**

(A) (i) $a_1 = 80 \times 10 = 800$ mm^2
$a_2 = 100 \times 10 = 1000$ mm^2



(ii) $x_1 = \dfrac{10}{2} = 5$ mm

$x_2 = \dfrac{100}{2} = 50$ mm

$y_1 = 10 + \dfrac{80}{2} = 10 + 40 = 50$ mm

$y_2 = \dfrac{10}{2} = 5$ mm

$\bar{x}_1 = \dfrac{a_1x_1 + a_2x_2}{a_1 + a_2} = \dfrac{(800 \times 5) + (1000 \times 50)}{(800 + 1000)}$

$\bar{x} = 30$ mm

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2} = \frac{(800 \times 50) + (1000 \times 5)}{(800 \times 1000)}$$

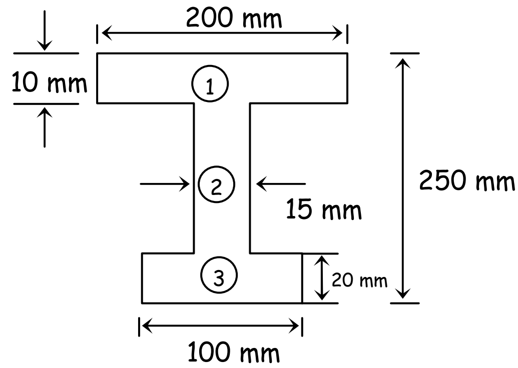
$\bar{y}$ = 25 mm

Q.6(b) Find the centroid of the I-section with following details. **[4]**

(i) Top flange = 200 mmx 10mm **(ii) Bottom flange = 100 mm × 20 mm**

(iii) Web thickness = 15 mm **(iv) over all depth = 250 mm**

(A) $\bar{x} = x_1 = x_2 = \frac{200}{2}$ = 100 mm



a_1 = 200 × 10 = 2000 mm^2

a_2 = 15 × 220 = 3300 mm^2

a_3 = 100 × 20 = 2000 mm^2

A = $a_1 + a_2 + a_3$

= 2000 + 3300 + 2000 = 7300 mm^2

y_1 = 20 + 220 + 10/2 = 245 mm

$y_2 = 20 + \frac{220}{2}$ = 130mm,

y_3 = 20/2 = 10 mm

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{A} = \frac{2000 \times 235 + 3300 \times 130 + 2000 \times 10}{7300}$$

$$\bar{y} = \frac{91900}{7300} = 125.89 \text{ mm.}$$

$\therefore \quad a(\bar{x}, \bar{y})$ = (100 mm, 125.89 mm)

Q.6(c) A wall of height 6m has one side vertical and other inclined. The top thickness is 1 m and bottom thickness is 4 m. Find its centroid. **[4]**

(A) From given data,

Divide the section of retaining wall into rectangle (1) and triangle (2) and taking the complete section of retaining wall in first quadrant

$A_1 = 1 \times 6 = 6\text{m}^2$

$A_2 = \frac{1}{2} \times 3 \times 6 = 9\text{m}^2$

$X_1 = \frac{1}{2} = 0.5\text{m}$

$X_2 = 1 + \frac{1}{3} \times 3 = 2$

= 2 m ... wrt to OY

$Y_2 = \frac{1}{3} \times 6 = 2\text{m}$ .... wrt to OX.

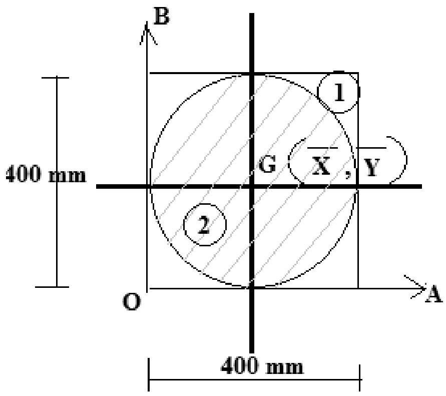
$$\bar{X} = \frac{A_1X_1 + A_2X_2}{A_1 + A_2} = \frac{(6 \times 0.5 + 9 \times 2)}{(6 + 9)} = 1.4\text{m}$$

$$\bar{y} = \frac{A_1Y_1 + A_2Y_2}{A_1 + A_2} = \frac{(6 \times 3 + 9 \times 2)}{(6 + 9)} = 2.4\text{m}$$

$G(\bar{x}, \bar{y}) = G(1.4\text{m}, 2.4\text{m})$

Q.6(d) A square of 400 mm side from which a circle of 400 mm diameter is cut-off from the centre. Find centroid of the remaining area. **[4]**

(A)



(i) Area calculation

$A_1 = 400 \times 400 = 160000\ mm^2$

$A_2 = (\pi / 4) \times (400)^2 = 125663.706\ mm^2$

$A = A_1 = A_2 = 34336.293\ mm^2$

(ii) Location of x^{-}

$x_1 = 400 / 2 = 200$ mm

$x_2 = 400 / 2 = 200$ mm

$$\bar{x} = \frac{A_1x_1 - A_2x_2}{A}$$

$\bar{x}$ = 200mm

(iii) Location of y^{-}

$y_1 = 400 / 2 = 200$ mm

$y_2 = 400 / 2 = 200$ mm

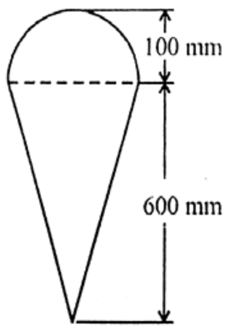
$$\bar{y} = \frac{A_1y_1 - A_2y_2}{A}$$

$\bar{y}$ = 200mm

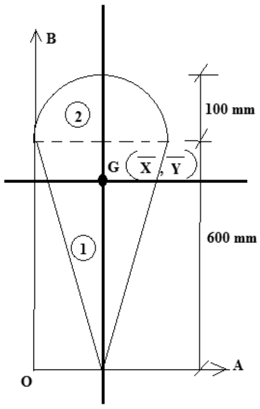
Hence, centroid (G) for given section lies at $G(\bar{x}, \bar{y})$

= (200 mm from oB and 200 mm from OA)

Q.6(e) Locate the position of centroid of an ice-cream cone as shown in figure. **[4]**



(A)



Note : Considering Centroid

(i) Figure is symmetric @ y – y axis and hence,

$\bar{x}$ = Maximum horizontal dimension / 2
= 200 / 2
= 100 mm

(ii) Area Calculation

$A_1 = \frac{1}{2}bh_1 = \frac{1}{2}\times 200\times 600$ = 60000mm^2

$A_2 = \frac{\pi r^2}{2} = \frac{\pi(100)^2}{2}$ = 15707.96mm^2

$A = A_1 + A_2$ = 75707.96mm^2

(iii) $\bar{y}$ calculation

$y_1 = \frac{2}{3}h_1 = \frac{2}{3}\times 600$ = 400 mm

$y_2 = h_1 + \frac{4r}{3\pi} = 600 + \left(\frac{4\times 100}{3\pi}\right)$ = 642.44mm

$\bar{y} = \frac{A_1y_1 + A_2y_2}{A}$

$\bar{y}$ = 450.30 mm

Hence, centroid (G) for given ice cream cone lies at $G(\bar{x}, \bar{y})$

= (100 mm from OB and 450.30 mm from OA)

OR

Note : Considering Center of Gravity of ice-cream cone.

(i) Figure is symmetric @ y-y axis and hence,

$\bar{x}$ = Maximum horizontal dimension / 2
= 200 / 2
= 100 mm

(ii) Volume Calculation

$V_1 = (1/3)\pi r_1^2 = (1/3)\pi(100)^2 \times 600 = 6.28318 \times 10^6$ mm^3

$V_2 = (2/3)\pi r_2^3 = (2/3)\pi(100)^3 = 2.094395 \times 10^6$ mm^3

$V = V^1 + V^2 = 8.377575 \times 10^6$ mm^3

(iii) $\bar{y}$ calculation

$y1 = h_1 - \frac{h_1}{4} = 600 - \frac{600}{4}$ = 450mm

$y2 = h_1 + \frac{3r_2}{8} = 600 + \left(\frac{3\times 100}{8}\right)$ = 637.5mm

$\bar{y} = \dfrac{V_1 y_1 + V_2 y_2}{V}$

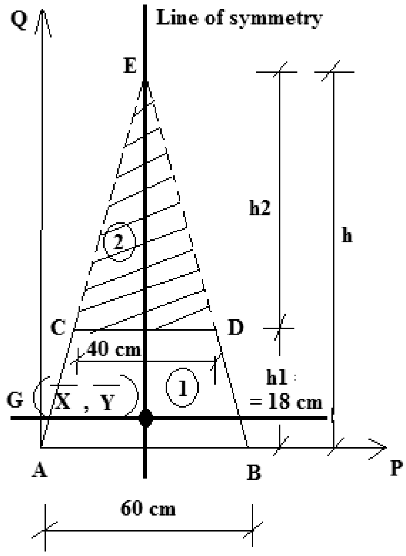
$\bar{y}$ = 496.875mm

Hence, Centre of Gravity (G) for given ice cream cone lies $G(\bar{x}, \bar{y})$

= (100 mm from OB and 496.875 mm from OA)

Q.6(f) The frustum of a cone has top diameter 40 cm and bottom diameter 60 cm with height 18 cm. Calculate $\bar{Y}$ only. **[4]**

(A)



Let, Full cone as figure 1 and cut cone as figure 2

(i) Figure is symmetric @ y – y axis and hence,

$\bar{x}$ = Maximum horizontal dimension / 2

= 60 / 2

= 30 cm

h_1 = 18 cm, h_2 = Height of cut cone

In triangle, ABE and CDE

$\dfrac{h}{60} = \dfrac{h_2}{40}$

$h = \dfrac{60}{40} h_2$

$h = 1.5h_2$

$h_1 + h_2 = h$

$h_1 + h_2 = 1.5h_2$

$h_1 = 1.5h_2 - h_2$

$h_1 = 0.5h_2$

$18 = 0.5h_2$

h_2 = 36cm

h = 18 + 36 = 54cm

(ii) Volume Calculation

$V_1 = (1/3)\pi r_1^2\ h = (1/3)\pi(30)^2 \times 54 = 50.86 \times 10^3\ cm^3$

$V_2 = (1/3)\pi r_2^2\ h+ = (1/3)\pi(20)^2 \times 36 = 15.07 \times 10^3\ cm^3$

$V = V_1 - V_2 = 35.82 \times 10\ cm^3$

(iii) $\bar{y}$ calculation

$$y_1 = \frac{h}{4} = \frac{54}{4} = 13.5\text{cm}$$

$$y_2 = h_1 + \frac{h_2}{4} = 18 + \left(\frac{36}{4}\right) = 27 \text{ cm}$$

$$\bar{y} = \frac{V_1 y_1 - V_2 y_2}{V}$$

$$\bar{y} = 7.815 \text{ cm}$$

Hence, centre of gravity (G) for given frustum of cone lies at $G(\bar{x}, \bar{y})$

= (30 cm from AQ and 7.815 cm from AP)

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