Theory of Machines

Time: 3 Hrs.] Prelim Question Paper Solution **[Marks : 70**

Q.1 Attempt any Five of the following: [10]

Q.1(a) List four inversion of Four bar Chain Mechanism [2]

Ans.: (i) Coupled wheel of locomotive
(ii) Pentograph
(iii) Beam engine
(iv) Watt straight line indicator mechanism

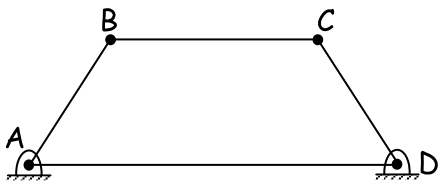
Q.1(b) Define kinematic link and kinematic chain. [2]

Ans.: **Kinematic link:** Each part of a machine, which moves relative to some other part, is known as a kinematic link (or simply link) or element.

Kinematic Chain: When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion (i.e. completely or successfully constrained motion), it is called a kinematic chain.

Q.1(c) State the necessity of Acceleration diagram of a mechanism. [2]

Ans.:



A, B C, D = Turning Pair

Acceleration in Mechanism:

Linear acceleration :

It is defined as the rate of change of velocity with respect to time. It is a vector quantity and denoted by a letter 'a'. Mathematically it is written as $a = \frac{dv}{dt}$ in m/sec^2.

Angular acceleration :

It is defined as the rate of change of angular velocity with respect to time. It is represented by a symbol 'α' and mathematically written as $\alpha = \frac{d\omega}{dt}$.

The acceleration of a particle whose velocity changes both in magnitude and direction at any instant it must posses two components of acceleration.

(i) The centripetal or radial component which is along the line joining the point and its center of rotation and directed towards the center of rotation, that is at right angle to the direction of velocity at that instant.

(ii) The tangential component which is in the direction perpendicular to this line or tangential to path of a point at this instant and in the same direction as the velocity at that point. Sense of the tangential acceleration may be the same as that of velocity or opposite to it.

Procedure for drawing the Acceleration Diagram of a Mechanism:

(i) Draw the configuration or space diagram with suitable scale.

(ii) Draw velocity diagram for the configuration by relative velocity method as explained in earlier section.

(iii) Find the velocity of various link in the configuration from velocity diagram and evaluate centripetal component of acceleration for various link as shown in table.

Link	Length of link	Velocity of link from velocity diagram	Centripetal acceleration $a^c = \dfrac{V^2}{(\text{Length of link})}$	Tangential acceleration $a^t = \alpha \times$ length of link

Q.1(d) State four applications of Cam and Followers. [2]

Ans.: Applications of Cam and Followers

(i) Cam and follower are widely used for operating inlet and exhaust valve of I.C. engine.
(ii) These are used in wall clock.
(iii) These are used in feed mechanism of automatic lathe machine.
(iv) These are used in paper cutting machine.
(v) Used in weaving textile machineries.

Q.1(e) State types of cams. [2]

Ans.: Types of cam : (i) Radial or disc cam, (ii) Cylindrical cam

Q.1(f) Define fluctuation of energy and coefficient of fluctuation of energy. [2]

Ans.: **(i) Fluctuation of energy:** The difference of maximum and minimum kinetic energy of flywheel is known as Fluctuation of energy

(ii) Coefficient of fluctuation of energy: It is defined as the ratio of the maximum fluctuation of energy to the work done per cycle.
It is denoted by $K_e = (E_1 - E_2)$ / work done per cycle

Q.1(g) State the adverse effect of imbalance of rotating elements of machine. [2]

Ans.: Adverse effect of imbalance of rotating elements. (Minimum two points)

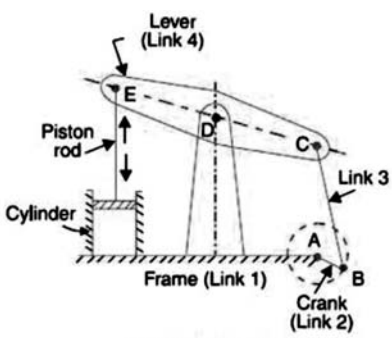
(i) Vibration, noise and discomfort,
(ii) Machine accuracy get disturbed,
(iii) Power losses,
(iv) More maintenance

Q.2Attempt any THREE of the following: [12]

Q.2(a) Draw a neat sketch and explain working of beam engine. [4]

Ans.: Beam engine (crank and lever mechanism)

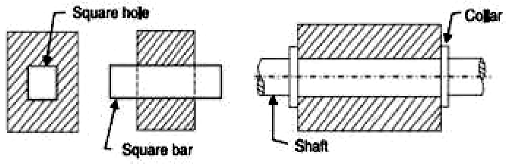
A part of the mechanism of a beam engine (also known as cranks and lever mechanism) which consists of four links is shown in figure. In this mechanism, when the crank rotates about the fixed centre. A, the lever oscillates about a fixed centre D. The end E of the lever CDE is connected to a piston rod which reciprocates due to the rotation of the crank. In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.



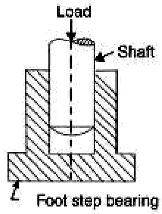
Beam Engine

Q.2(b) Define completely constrained motion and successfully constrained motion with neat sketch. State one example of each. [4]

Ans.: Completely constrained motion. When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion. For example, the piston and cylinder (in a steam engine) form a pair and the motion of the piston is limited to a definite direction (i.e. it will only reciprocate) relative to the cylinder irrespective of the direction of motion of the crank, e.g. Square bar in a square hole. & Shaft with collars in a circular hole.



Completely constrained motion.



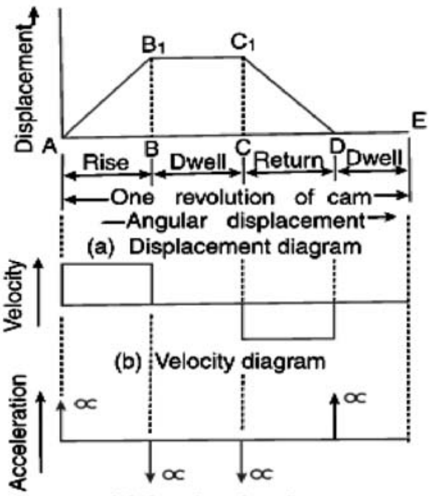
Successfully constrained motion

Successfully constrained motion. When the motion between the elements, forming a pair, is such that the constrained motion is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion. Consider a shaft in a foot-step bearing as shown in figure.

The shaft may rotate in a bearing or it may move upwards. This is a case of incompletely constrained motion. But if the load is placed on the shaft to prevent axial upward movement of the shaft, then the motion of the pair is said to be successfully constrained motion. The motion of an I.C. engine valve (these are kept on their seat by a spring) and the piston reciprocating inside an engine cylinder are also the examples of successfully constrained motion.

Q.2(c) Draw the labelled displacement, velocity and acceleration diagrams for a follower when it moves with uniform velocity. [4]

Ans.:



(c) Acceleration diagram

Q.2(d) The central distance two shaft is 4 m having two pulleys with diameter having 500 mm and 700 mm respectively, find the length of belt required : [4]
(i) for open belt drive (ii) for cross belt drive

Ans.: Central distance between two shafts; C = 4 Meters; = 4000 mm
Smaller pulley diameter = d = 500 mm; Smaller pulley radius = r = 250 mm;
Larger pulley diameter = d = 700 mm; larger pulley radius = r = 350 mm;

Angle subtended by each tangent β

(a) Length of open belt drive

Angle subtended by each tangent β $= \sin^{-1}(R - r/C) = \sin^{-1}((350-250)/4000)$

$B = 0.025$ radians

$L_O = \pi (R + r)\ 2 \times \beta (R - r) + 2\ C \times \cos\beta = 9.889$ m

$L_O = 9.889$ m

(b) Length of cross belt drive

Angle subtended by each tangent $\beta = \sin^{-1} (R + r/C) = \sin^{-1}(350 + 250) /4000$

$\beta = 0.01575$ radians.

$L_C = \pi(R + r)^2 \times \beta(R - r) + 2C \times \cos \beta - 9.903$ m

$L_O = 9.903$ m

Q.3Attempt any THREE of the following: [12]

Q.3(a) What is machine? Differentiate between a machine and a structure. [4]

Ans.: **Machine definition:** A device which transforms available energy into useful work is called as machine.

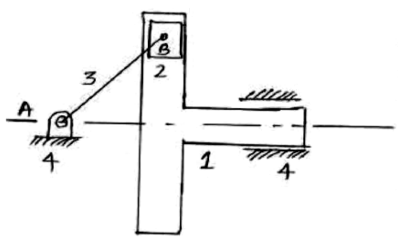
Sr. No.	Machine	Structure
(1)	Relative motion exist between its parts.	No relative motion exists between its members.
(2)	Links are meant to transmit motion and forces which are dynamic (both static and kinetic)	Members are meant for carrying loads subjected to forces having straining actions.
(3)	Machines serve to modify and transmit mechanical work.	Structure serves to modify and transmit forces only.
(4)	Example: shaper, lathe, screw jack etc.	Examples: roof trusses, bridges, buildings, machine frames etc.

Q.3(b) Draw the neat sketch of 'Scotch yoke Mechanism'. [4]

Ans.: In scotch yoke mechanism slide block of first inversion is fixed. End B of crank (link 3) rotates about center A. Link 1 reciprocates in horizontal direction.

Sliding pair-two ---1-4, 2-1

Turning pair-two ---2-3, 3-4



Scotch Yoke Mechanism

Q.3(c) Explain the principle of working of Internal Expanding Brake using neat sketch. [4]

Ans.: **Internal Expanding Brake**

- The brake drum encloses the complete brake mechanical and protect it from dust and moisture. The inner side of drum is open. The backing plate at the open side of the brake drum complete the brake enclosure and hold the brake assembly. The backing plate is attached to the vehicle axle housing and acts as a frame. For fastening the brake shoe and operate the cam mechanism with it linkage. The wheel attaching bolts on the brake drum connects the wheel and drum.

* It consist of two semi-circular brake shoe having friction lining on their outer surface. The brake shoe are hinged to back plate at lower end by an anchor pin while other end rest on cam or toggle.
* This cam can be turned or actuated by camshaft passes through hole in back plate. The camshaft can be operated by brake pedal through linkage.
* When the brake pedal is pressed, the cam turns by expanding the brake shoe outward against the retractor spring force. The friction lining comes in contact with the drum causes friction between them.
* This force of friction opposes the direction of motion and reduce the speed or stop the vehicle. When brake pedal is released, the retracting spring pull the brake shoe inward which turn the cam and brakes are released.

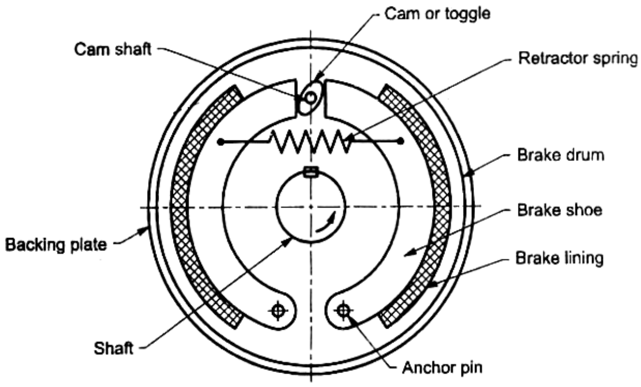
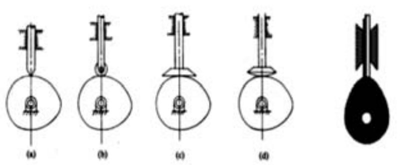
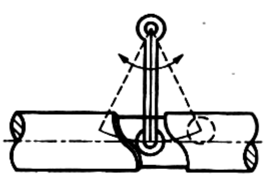
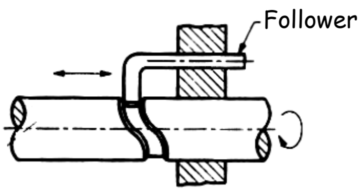


Fig. : Internal expanding

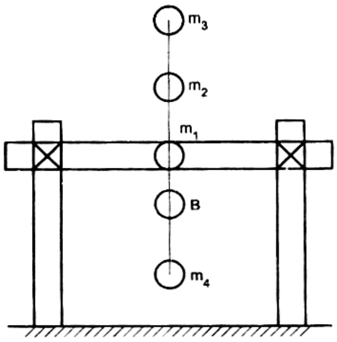
Q.3(d) Distinguish between Radial and Cylindrical Cam. Also draw the sketches of both the cams. **[4]**

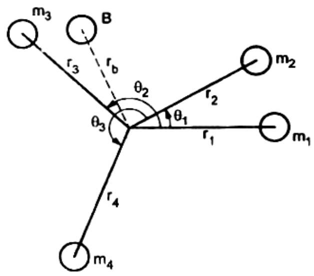
Ans.:

Sr. No.	**Radial Cam**	**Cylindrical Cam**
(i)	In radial cam, follower movement takes place perpendicular to axis of rotation of cam	In Cylindrical cam, follower movement takes place parallel to axis of rotation of cam
(ii)	Periphery of disc is working surface	Periphery of cylinder is working surface
(iii)	Spring or sraving is required bring follower to its initial position.	Spring is not required.
(iv)	Diagram: 	Diagram:  (a) Oscillating motion  (b) Reciprocating motion

Q.3(e) Explain the method of balancing of different masses revolving in the same plane. [4]

Ans.: Balancing of Several Masses Rotating in the Same Plane





(a) Fig. : Space diagram **(b)**

- Consider a number of masses of magnitude m_1, m_2, m_3 and m_4 (say four) are rotating at a radius of r_1, r_2, r_3 and r_4 from the axis of rotating shaft. Let θ_1, θ_2, θ_3, and θ_4 be the angles made of these masses with the horizontal line as shown in figure.

  When such a system revolve it is subjected to a concurrent system of centrifugal forces acting radially outward through each mass center. The resultant of these system can be found by,

  (i) Graphically – as discussed below:

  (ii) Analytically

Drawing of forces (Vector diagram)

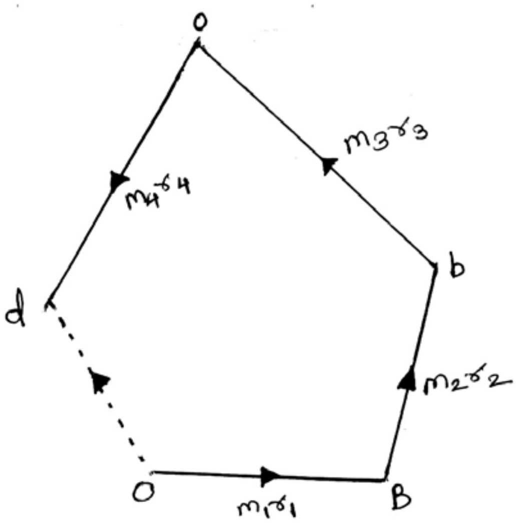


Fig.: Vector diagram

The magnitude and position of the balancing mass can be found as:

- First draw space diagram with given position of masses as shown in figure.
- Find the value of centrifugal force (m_1r_1) or (m_2r_2) exerted on the each rotating mass.
- Now draw the vector diagram with the help of centrifugal force such that ab represent the centrifugal force exerted on mass m1 (or m_1 r_1) with magnitude and direction with suitable scale. Similarly construct, bc, cd.
- Now, according to law of polygon, the closing side 'od' represent the resultant force and direction of balance mass B but opposite in direction.
- Shift this resultant in the space diagram to find the angle of balance mass with horizontal.

Analytical Method:

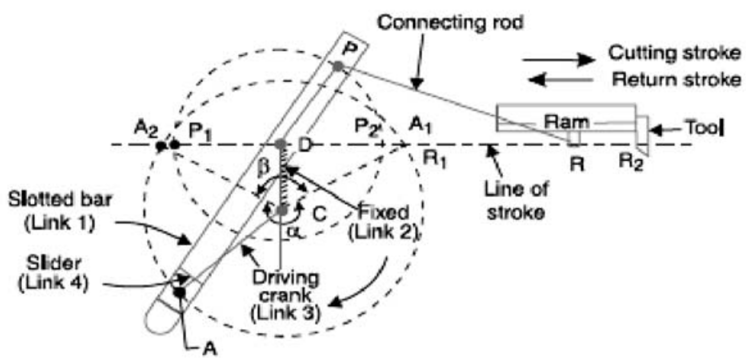
The magnitude and the direction of the balancing mass may be obtained, analytically as discussed below.

- Find centrifugal force exerted by each mass on the rotating shaft. (centrifugal force is the product of mass and its respective radius of rotation).
- Resolve the centrifugal forces horizontally (ΣH) and vertically (ΣV).
  Sum of horizontal component of the centrifugal forces
  $\Sigma H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + ...$
  and sum of vertical component of the centrifugal forces.
  $\Sigma V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + ...$
- The magnitude of the resultant centrifugal force
  $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$
- The resultant force makes an angle θ_R with the horizontal, $\tan\theta = \frac{\Sigma H}{\Sigma V}$
- The balance force is then equal to the resultant force but in opposite direction.
- To find the magnitude of the balance mass,
  $R_b = m_b r_b$
  Where, m_b = Balancing mass,
  r_b = radius of rotation of balancing mass.

Q.4Attempt any TWO of the following: [12]

Q.4(a) Draw the construction of 'Whitworth Quick Return mechanism'. [6]

Ans.: **Whitworth quick return motion mechanism.** This mechanism is mostly used in shaping and slotting machines. In this mechanism, the link CD (link 2) forming the turning pair is fixed, as shown in figure. The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank CA (link3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at A slides along the slotted bar PA (link 1) which oscillates at a pivoted point D. The connecting rod PR carries the ram at R to which a cutting tool is fixed. The motion of the tool is constrained along the line RD produced, i.e. along a line passing through D and perpendicular to CD.



When the driving crank CA moves from the position CA1 to CA2 (or the link DP from the position DP1 to DP2) through an angle α in the clockwise direction, the tool moves from the left hand end of its storke to the right hand end through a distance 2 PD.

Now when the driving crank moves from the position CA2 to CA1 (or the link DP from DP2 to DP1) through an angle β in the clockwise direction, the tool moves back from right hand end of its stroke to the left hand end.

A little consideration will show that the time taken during the left to right movement of the ram (i.e. during forward or cutting stroke) will be equal to the time taken by the driving crank to move from CA1 to CA2. Similarly, the time taken during the right to left movement of the ram (or during the idle or return stroke) will be equal to the time taken by the driving crank to move form CA2 to CA1.

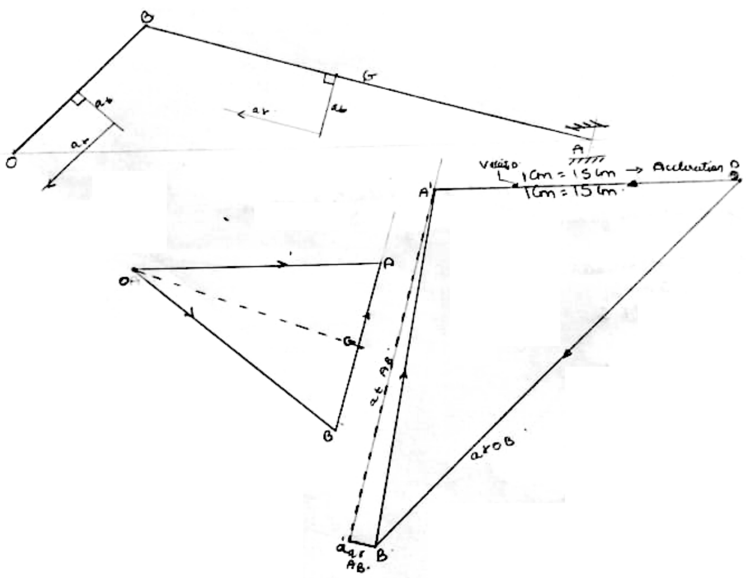
Since the crank link *CA* rotates at uniform angular velocity therefore time taken during the cutting stroke (or forward stroke) is more than the time taken during the return stroke. In other words, the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke.

The ratio between the time taken during the cutting and return strokes is given by

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360^\circ - \alpha} \text{ or } \frac{360^\circ - \beta}{\beta}$$

Q.4(b) In a Single slider crank mechanism, crank OB = 50mm, the length of connecting rod AB = 125 mm. The point '*G*' is at 60 mm form point 'B'. Crank OB is rotated at 45° from OA. The Crank rotates at 200 rpm, find out the velocity of point '*G*' and angular acceleration of AB. **[6]**

Ans.:



Crank = 50mm = 5cm

Connecting rod = 125 mm → 12.5 cm

$w = \frac{2\pi N}{60} \frac{2\pi \times 200}{60}$

w = 20.94

$\therefore w_{AB}$ =20.94

$V_{AB} = L_{AB} \times W_{AB}$

=0.05 × 20.94

= 1.047 m/s

or

$V_{AB} = W_{ABXLAB}$

=20.94 × 5cm

V_{AB} =104.7 cm/s

1.047 m/s

V_{OB} =(OB)× Scale

6.9 × 15

103.5 cm/s

1.035

V_{OA} = L(OA)× Scale

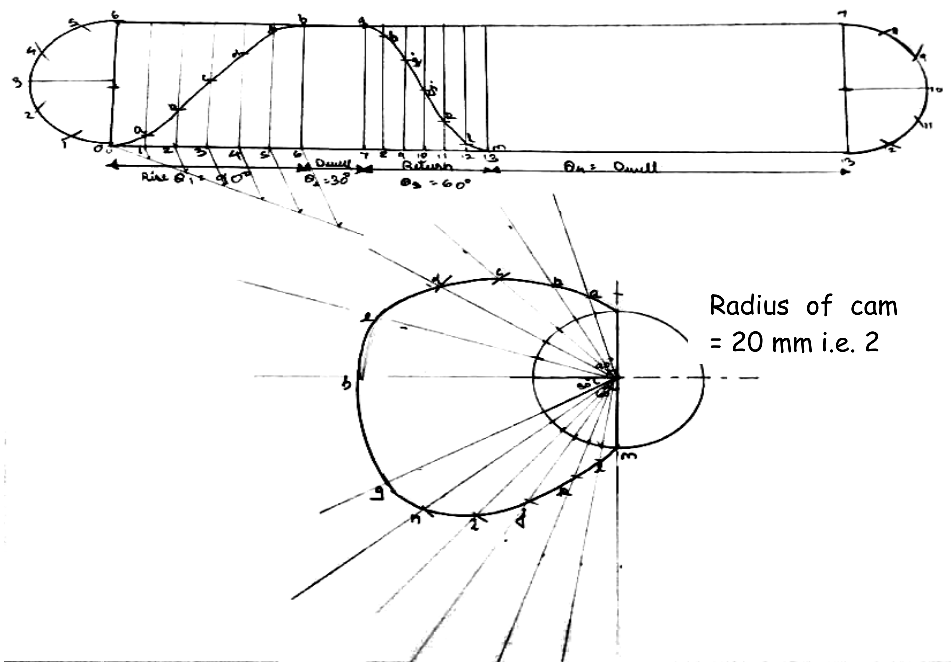
= 6.5 × 15

=97.5 cm/s

Sr. no.	Name as Link	L	V	$a_r = \frac{v^e}{L}$	at	
1	OB	0.005	1.035	214.24	–	14.28
2	BA	0.125	1.047	8.76	0	–0.6
3	OA	–				

Q.4(c) A cam is to be designed for knife edge follower with following data. Cam lift 40 mm during 90°of cam rotation with SHM, Dwell for 30°, during return stroke 60° of cam rotation by SHM and remaining is for dwell. Draw profile of cam **[6]**

Ans.:



Q.5 Attempt any TWO of the following: **[12]**

Q.5(a) A leather belt is required to transmit 7.5 kW from a pulley 1.2 m in diameter running at 250 rpm. The angle of contact is 165° and the co-efficient of friction between the belt and the pulley is 0.35. If the safe working stress for the leather belt is 2 MPa, density of leather is 1050 kg/m^3 and the thickness of belt is 10 mm, determine the width of belt, taking centrifugal tension into account. **[6]**

Ans.: We know that velocity of the belt

$$V = \frac{\pi . d . N}{60} = \frac{\pi \times 1.2 \times 250}{60} = 15.71 \text{ m/s}$$

and power transmitted (P)

$P = (T_1 - T_2)V$

$7.5 \times 10^3 = (T_1 - T_2)\ 15.71$

$\therefore T_1 - T_2 = 7500 / 15.71 = 477.4$ N ... (i)

We know that

$$\frac{T_1}{T_2} = e^{\mu\theta} \qquad \therefore \frac{T_1}{T_2} = e^{0.3 \times 165 \times \frac{\pi}{180}} \qquad \therefore \frac{T_1}{T_2} = 2.375 \qquad \text{... (ii)}$$

from equation (i) and (ii)

$T_1 = 824.6$ N and $T_2 = 347.2$ N

We know that mass of the belt per meter length,

m = Area × length × density = bt □ R

= b × 0.01 × 1 × 1050 = 10.5 b kg

∴ centrifugal Tension,

$T_C = m.v^2 = 10.5\ b\ (15.71)^2 = 2591.44b$ N and Maximum Tension in the belt,

$T = \sigma.\, b.t = 2 \times 10^6 \times b \times 0.01$

$= 20000\ b$ N

We know that,

$T = T_1 + T_C$

$\therefore$ 20000 b = 824.6 + 2591.44 b

$\therefore$ 20000 b – 2591.44b = 824.6

$\therefore$ 17408.56b = 824.6

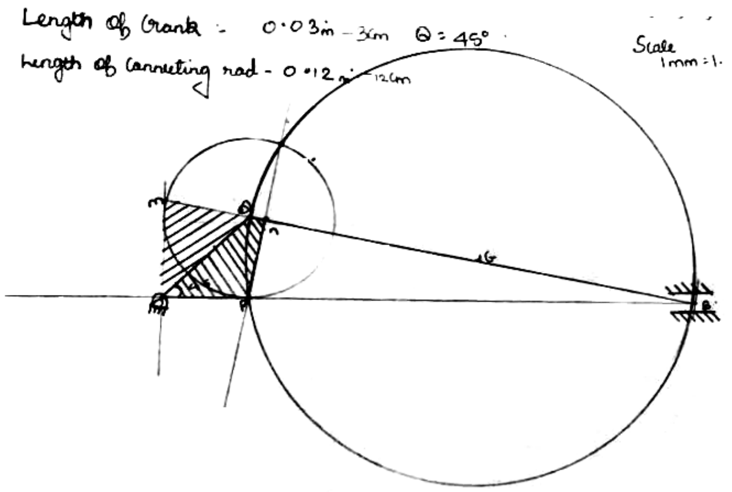
$$\therefore b = \frac{824.6}{17408.56}$$

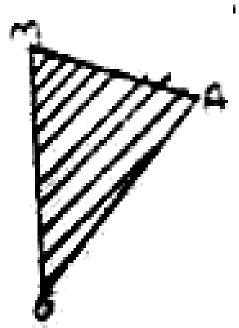
$\therefore$ b = 0.0473m

$\simeq$ 47.37m

Q.5(b) In a slider crank mechanism, lengths of crank and connecting rod are 30mm and 120mm respectively. The crank rotates at 180 rpm clockwise. When the crank rotates to 45° from Inner Dead Centre, find the velocity and acceleration of Slider using Klein's construction. Also find angular velocity and acceleration of connecting rod. **[6]**

Ans.:





OA – Angular velocity of crank

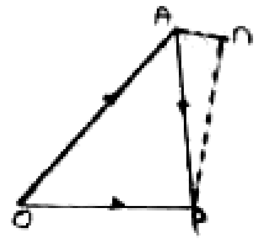
AN – Angular velocity of crank–RoD

OM – Angular velocity of slider

V_{AB} = L[AM] × Scale

= 2.1 × 1

= 2.1



a_{OA} = L(OA) × Scale

Acceleration of Connecting road a_{AB} = L(AD) × Scale

= 2.1 × 1

= 2.1

$$\frac{BG}{BA} = \frac{\log}{\text{lba}}$$

$$\frac{0.06}{0.125} = \frac{\text{lbg}}{4.8}$$

= 2.30

If point G is at an distance of 2.3 cm from point B in velocity diagram

l(OG) : 6.3 from velocity Diagram

$\therefore V_{OG} = l(OG) \times \text{Scale}$
$= 6.3 \times 15$
$= 94.5$

Angular Acceleration of connecting rod:
a'A = 10m

$a_{BA} = 10 \times \text{scale}$
$= 10 \times 15 = 150 \text{ m/ Su}^2$
$a_{AB} = d_{AB} \times AB$

$a_{AB} = \frac{a_{AB}}{AB} = \frac{150}{0.125} = 1200 \text{ rad/sec}$

Q.5(c) Draw neat labeled sketch of Hartnell Governor and explain its working. [6]

Ans.: Hartnell Governor

- It is a spring loaded governor as shown in figure. It consist of two bell crank levers pivoted at the point O and 0 to the frame. The frame is attached to the governor spindle and therefore rotate with it.
- Each bell crank lever carries a ball at the end of the vertical arm OB and rollar at the horizontal arm OR.
- A compressive helical spring provides equal downward force on the two rollers through a collar on the sleeve. The spring force may be adjusted by screwing a nut up or down on the sleeve.
- When engine speed increases (load decreases) the governor speed increase so flyball moves outward due to centrifugal force and sleeve moves upward by compressing the spring and decreases the fuel supply to engine and vice versa.
- The function of helical spring in the hartnell governor is to provide equal downward forces on the two rollers through a collar on the sleeve.

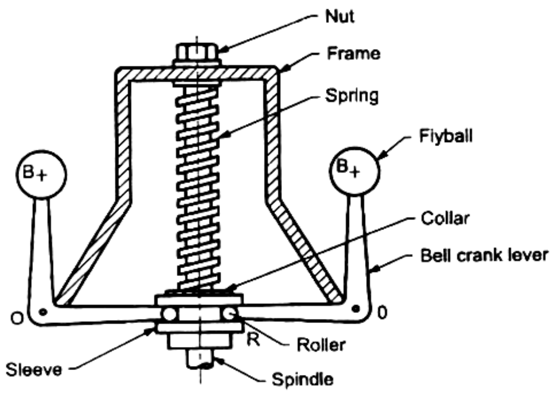


Fig. : Hartnell

Q.6Attempt any TWO of the following: [12]

Q.6(a) (i) State types of gear train and explain any one. [6]

(ii) Draw turning moment diagram for single cylinder four stroke I.C. Engine.

Ans.: (i) Types of gear trains

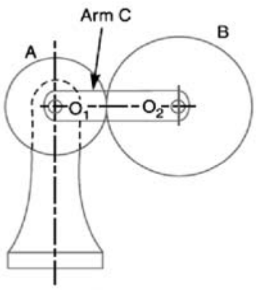
- Simple gear train
- Compound gear train
- Epicyclic gear train
- Inverted gear train

Simple gear train:
When there is only one gear on each shaft, it is known as simple gear train. The gears are represented by their pitch circles. When the distance between the two shafts is small, the two gears are made to mesh with each other to transmit motion from one shaft to the other

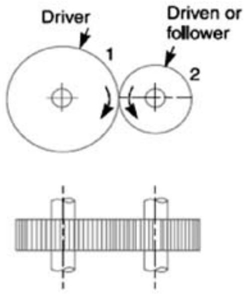
Epicyclic gear train:
A simple epicyclic gear train is shown in figure where a gear A and the C have a common axis at O_1 about which they can rotate. The gear B meshes with gear A and has its axis on the arm at O_2, about which the gear B can rotate. If the arm is fixed, the gear train is simple and gear A can drive gear B or vice-versa, but if gear A is fixed and the arm is rotated about the axis of gear A (i.e. O1), then the gear B is forced to rotate upon and around gear A. Such a motion is called epicyclic and the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as epicyclic gear trains.

Compound Gear Train
When there are more than one gear on a shaft, it is called a compound train of gear. Whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is intensified by providing compound gears on intermediate shafts.

Reverted Gear Train
When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as reverted gear train. We see that gear 1 (i.e. first driver) drives the gear 2 (i.e. first driven or follower) in the opposite direction.

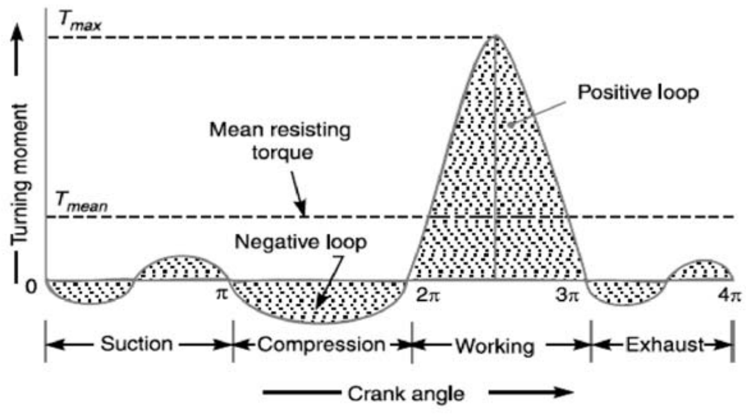


Epicyclic gear train.



Simple gear train.

Turning Moment Diagram



Turning moment diagram for a four stroke cycle internal combustion engine.

Q.6(b) Draw neat labeled sketch of Diaphragm Clutch and explain its working. [6]

Ans.: Diaphragm Clutch

- In this clutch, in order to produce the required pressure for engaging it, a form of diaphragm or conical spring is employed in place of coil spring.
- This type of clutch is quite advantages because it requires no release levers and the spring itself acts as a series of levers.
- Some of the conical springs used even are not having the constant-rate characteristics.
- The pressure of spring is always varying.
- It increases till the spring reaches to its flat position and decreases with the passing of this position.
- In case of this clutch, the driver has not to exert as high a pedal pressure to hold the clutch out of engagement as in case of a coil spring type.
- In case of a coil spring type of clutch, when the pedal is depressed to disengage the clutch, the spring pressure further increases.
- In figure is shown in the tapered finger type of diaphragm spring operating in a clutch assembly.
- The spring is pivoting on the rear rings in its engaged position while holding itself on the clutch cover.

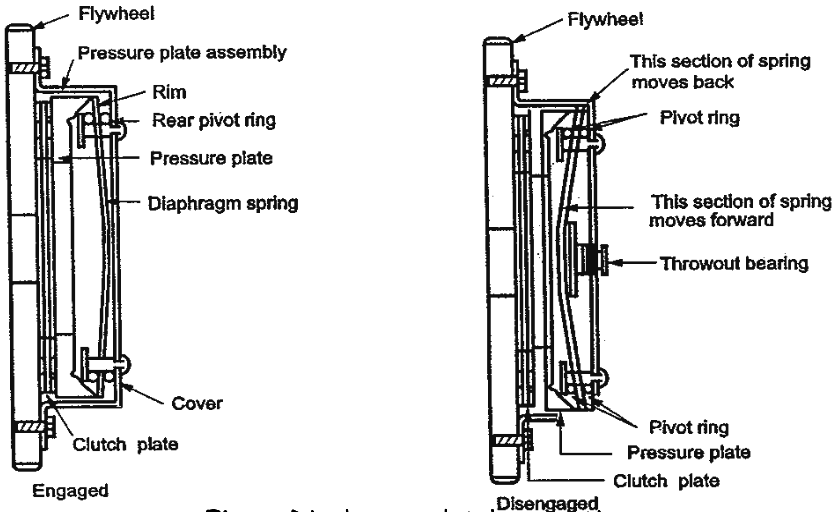
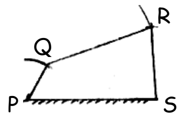


Fig. : Diaphragm clutch operation

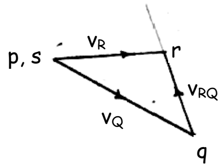
- In this position the pressure plate is in contact with its outer rim. Therefore, sufficient pressure is exerted by the spring making a firm between the pressure plate and clutch plate as well as the flywheel in this natural conical position.
- Now in order to disengage the clutch, the pedal is depressed.
- The throughout bearing is moved toward the flywheel by the linkage due to pedal depression.
- As the spring is pivoting on the front pivot ring, therefore, the bearing contacting the inner portion of the conical spring moves that portion forward resulting the rim to move backward.
- The clutch disc gets released from the contact with both the driving members because the pressure on the pressure plate is removed.
- The release levers would move towards the withdrawn bearing and prevent the clutch from engaging fully due to wearing down of the friction lining.
- To prevent the clutch slipping, a free movement of about 25 mm at the clutch pedal or 1.5 mm at the withdrawal bearing is provided.

Q.6(c) PQRS is a four bar chain with link PS fixed. The lengths of links are PQ = 62.5 mm, QR = 175 mm, RS = 112.5 mm and PS = 200 mm. The crank PQ rotates at 10 rad/sec clockwise. Draw velocity and acceleration diagram, when angle QPS = 60° and Q and R lie on the same side of PS. Find the angular velocity and angular acceleration of links QR and RS. **[6]**

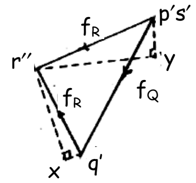
Ans.:



Space diagram



Velocity diagram



Acceleration diagram

Calculations :

$V_{QP} = \omega_{qp} \times PQ = 10 \times 0.0625 = 0.625$ m/s

From Velocity diagram,

By measurement, $V_{RQ} = 0.333$ m/s,

$\omega_{QR}/RQ = 0.333/0.175 = 1.9$ rad/s (Anti clockwise)

By measurement, $V_{RS} = 0.426$ m/s,

$\omega_{RS} = V_{RS}/SR = 0.426/01125 = 3.78$ rad/s (clockwise)

Find out radial acceleration of each link by using formula – V^2/length of link

$Fr_{QP} = 6.25$ m/s^2; $Fr_{RQ} = 0.634$ m/s^2; $Fr_{RS} = 1.613$ m/s^2

From acceleration diagram, measure all tangential components (ft)

Angular acceleration of link QR, α_{QR} = ft RQ/QR = 4.1/0.175

= 23.43 rad/s^2 (Anti clockwise)

Angular acceleration of link RS, α_{RS} = ft RS/SR = 5.3/0.1125

= 47.1 rad/s^2 (Anti clockwise)

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